Social Discounting and the Social Risk Premium with Public and Private Project Benefits

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Abstract

Public investment decisions are important firstly because the tax-payer's money should be wisely spent, secondly since the government can use its large market share to potentially steer the economy towards goals such as sustainability, and finally because questions of equality can be addressed in public investments. Rational investment decisions should aim to maximise societal welfare, and this can be best achieved in a second best world by using Cost-Benefit Analysis.

The choice of a Social Discount Rate to use in Cost-Benefit Analysis has long been a topic in economics. The most used choice rule for the discount rate is the Ramsey equation, which is derived formally in the thesis. To account for uncertainty, a social risk premium, instead of certainty equivalents is proposed to be used in Cost-Benefit Analysis. An expression for the risk premium is formally derived in the Ramsey equation.

To examine the need for a risk premium in the Social Discount Rate, the Arrow-Lind theorem is first derived to show, that by risk pooling the public investor can forgo the risk premium. Then, two extensions are integrated to the basic model, wherein first, the effects of an environmental externality, introduced as a 'public bad', and second, the effects of varying degrees of benefit rivalry under different risk aversion profiles, are examined.

The Arrow-Lind theorem is shown to hold in the 'public bad' case, if Hicks-Kaldor compensations were actually carried out. In the second case, the theorem is shown to hold, even with private project benefits, if individuals' relative risk aversion and relative prudence are low enough.

If the society's risk aversion profiles can be adequately estimated and they fulfil the criteria defined in the thesis, then the use of a risk premium in the discount rate can be justifiably given up. Finally, the thesis concludes, that different choice rules for the Social Discount Rate should be used based on the individual characteristics of the investment under consideration.

Keywords
discounting
social discount rate
social risk premium
public goods
public bads
cost-benefit analysis
uncertainty

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Tiivistelmä

Julkiset investointipäätökset ovat tärkeitä ensinnäkin siksi, että veronmaksajien rahoja tulisi käyttää järkevästi, toiseksi siksi, että valtio voi käyttää suurta markkinaosuuttaan ohjatakseen taloutta kohti tavoitteita kuten kestävyys, ja kolmanneksi siksi, että tasa-arvoon liittyviä kysymyksiä voidaan huomioida julkisissa investoinneissa. Rationaalinen investointipäätös tähtää yhteiskunnallisen hyvinvoinnin maksimointiin ja second best –maailmassa tämän saavuttaa parhaiten kustannus-hyötyanalyysi.

Yhteiskunnallisen diskonttokoron valinta kustannus-hyötyanalyysia varten on pitkään ollut talousteoreettisen keskustelun aiheena. Eniten käytetty sääntö sen valintaan on Ramseyn yhtälö, joka johdetaan muodollisesti tutkielmassa. Epävarmuuden huomioon ottamiseksi tutkielmassa ehdotetaan yhteiskunnallisen riskipreemion käyttämistä varmuusekvivalenttien sijaan. Matemaattinen muoto riskipreemiolle johdetaan osaksi Ramseyn yhtälöä.

Yhteiskunnallisen riskipreemion tarpeellisuuden tarkastelemiseksi, johdetaan tutkielmassa ensin Arrow-Lind -teoreema, joka osoittaa että väestön määrän ollessa riittävän suuri voidaan riskipreemio sivuuttaa. Alkuperäiseen malliin liitetään sitten kaksi laajennusta, joista ensimmäisessä ympäristöulkoisvaikutusten, jotka tulkitaan julkishaitakkeeksi, vaikutuksia ja jälkimmäisessä eri asteisen hyötyjen kilpailullisuuden vaikutuksia eri riskinkaihtamisprofiilien vallitessa tarkastellaan.

Arrow-Lind –teoreeman tulosten näytetään pitävän julkishaitakkeen tapauksessa, jos Hicks-Kaldor hyvitykset todella maksetaan. Jälkimmäisessä tapauksessa teoreeman tulokset pitävät, jopa yksityishyödykkeen omaisten projektin hyötyjen tapauksessa, jos yksilöiden suhteellisen riskinkarttamisen aste sekä suhteellinen varovaisuus ovat tarpeeksi alhaiset.

Jos yhteiskunnan riskinkaihtamisprofiileja on mahdollista tarpeeksi tarkasti arvioida ja tämä täyttäisi tutkielmassa määritellyt kriteerit, voidaan yhteiskunnallista riskipreemiota perustellusti jättää käyttämättä. Lopuksi, tutkielmassa todetaan, että erilaisia yhteiskunnallisen diskonttokoron valintasääntöjä tulisi käyttää harkinnan mukaisesti tapauskohtaisesti.

Avainsanat
diskonttaus
yhteiskunnallinen diskonttokorko
yhteiskunnallinen riskipreemio
julkishyödykkeet
julkishaitakkeet
kustannus-hyötyanalyysi
epävarmuus

Contents

List of Symbols

A	Random variable denoting the consumer's disposable income
α	The degree of rivalry of the benefits of the project, $0 \leq \alpha \leq 1$
B	A function depicting the net benefit from the project to each consumer
c	Consumption
$C(m)$	Environmental externality, a 'public bad', made up of a certain part and a mean-zero random effect
δ	Pure rate of time-preference
ε	A mean-zero random element of the consumer's disposable income
g	Growth rate of per capita consumption
γ	Elasticity of marginal utility of consumption/relative risk aversion
H	$H(\theta) = v' \cdot \theta$
$k(n)$	Consumer's risk premium for the public investment
$l(m)$	Risk premium of the sufferer of the externality
m	The number of consumers that suffer from the externality
n	Number of consumers/taxpayers
ω	$\omega = 1 - \alpha(1 - n)/n$
P	Index of relative prudence
$\hat{P}$	Index of partial relative prudence

ϕ Hicks-Kaldor compensation from the beneficiaries of the investment to the sufferers of the externality

ψ Prudence premium

ρ The social discount rate

s Each consumer's share of the public investment

t Period of time, t=*0,...,T-1*

θ Returns from the public investment, made up of a certain part $E(\theta)=\bar{\theta}$ and a mean zero random effect X

u The consumer's von Neumann-Morgenstern utility function

w Welfare function of the consumer

$\tilde{W}$ $\tilde{W}$=A-ξ/n

ξ Total cost of the project

Chapter 1

Introduction

The choice of a Social Discount Rate, to be used in public investments, might seem like a dry or exceedingly theoretical topic of discussion. Altering the discount rate, however, can have drastic impacts on the results of a study on the profitability of any given project, especially in the case of long term investments, such as climate policy or large infrastructure. The correct way of choosing a Social Discount Rate has remained the topic of academic discussions from the days of Ramsey (1928) and Pigou (1962), through the 1960s and 1970s (Sen, 1967; Marglin, 1969; Arrow and Lind, 1970), on to modern economists such as Gollier (2010), Weitzman (2010), and Dietz et al. (2015). On a less theoretical level, the choice of a suitable discount rate has been of increasing interest in public discussion during the past decades, as is evident from, say, the discussion sparked by the Stern Review (Stern et al., 2006). One of the main contributors to the discussion, Martin L. Weitzman, thus aptly characterises the choice of a Social Discount Rate as "the perennial dilemma" (Weitzman, 2001).

From an economist's perspective, public investment decisions should aim to maximise social welfare. From a theoretical point of view, and in a first best world[1], the way to achieve this would be to use a social welfare function, which is, in a utilitarian setting, an aggregation of the individual utility functions of the individuals in the society, and which thus represents the society's preferences (Jehle and Reny, 2011). The empirical estimation of the shape of a societal

[1] I.e. with a perfectly functioning economy.

welfare functions in different investment situations in the second best world we live in is, however, very difficult, if not impossible (Tuomala, 2009). A more easily implemented alternative that economic theory has offered for making economically rational choices, which aim at maximising societal utility in public investment decisions, is called Cost-Benefit Analysis.

Investment decisions under uncertainty have been the topic of a plethora of economic papers, especially during the last three decades. The booming of the financial sector accompanied by the ever growing computational powers of computers has led to more and more sophisticated theoretical models and decision-making methods. However, public investment decisions have seen remarkably little attention. This can, of course, be partly because economic theory already offers a 'ready' set of theoretical tools in Cost-Benefit Analysis. The theoretical body of Cost-Benefit Analysis is, nowadays, well formed, but not without its problems. The three main, partially intertwined, problems in Cost-Benefit Analysis that are addressed in this study are those of social discounting, coping with uncertainty, and accounting for externalities.

Given the often long time scales of public investment projects, logically the costs and benefits occuring at different time periods should be made commensurable. This is done by discounting, which, in turn, gives rise to the need for a correct Social Discount Rate. The choice of discount rate for public projects, unlike public investment decisions per se, has remained a relevant topic in economic theory, as previously mentioned. There is no unanimity regarding the choice of a Social Discount Rate. However, the so called Ramsey equation (Ramsey, 1928) has fixed its place as the most widely used definition of a Social Discount Rate, and is used, for example, as the basis of the recent survey by Drupp et al. (2015). The Ramsey equation defines the Social Discount Rate as a function of the social time-preference rate, the growth rate of per capita consumption and relative risk aversion.

The second problem, uncertainty is defined in my thesis, following Pearce (1983), as a situation where the probability distribution of possible costs and benefits of a project is unknown, *a priori*. In Cost-Benefit Analysis, the optimal approach to coping with uncertainty would be to estimate the probability distributions either based on previous experience or on the opinions of experts. A problem with using the expected values of costs and benefits given the probability distribution that is derived, is that they do not account for the level of risk aversion

Country\Year	2010	2011	2012	2013	2014
Finland	3.7%	3.8%	4.0%	4.1%	4.1%
Sweden	4.5%	4.4%	4.6%	4.5%	4.5%
UK	3.2%	3.0%	2.8%	2.6%	2.7%
USA	4.1%	3.9%	3.6%	3.4%	3.3%

Table 1.1: Public investments as a %-share of GDP, 2010-2014 (source: Eurostat, OECD.Stat)

the individuals in the society exhibit. This can be remedied by using expected utility instead of expected values, though here again problems arise given the difficulty of estimating the utility functions of individuals. Theory suggests using expected utility and the calculation of so called certainty equivalent values, to be used in Cost-Benefit Analysis. An alternative to calculating certainty equivalents is the introduction of a societal risk premium into the Social Discount Rate.

This choice of a Social Discount Rate and different approaches to coping with uncertainty or externalities make for good topics in academic papers, but do the answers to these questions matter in practice? I argue, that public investment decisions do matter, for a number of reasons. Firstly, quite self-evidently the tax-payer's money should be wisely spent. Secondly, governments can use their large market shares to steer the economy towards goals besides economic efficiency, by the inclusion of other than purely monetary criteria in investment decisions, for example, towards a more ecologically sustainable path. Since, if the government makes up a adequately large proportion of any given market, it can either set precedents or exploit its monopsony power when the objects of investment are partially privately produced. Finally, it is in public investments, more than elsewhere, that questions of equality and intergenerational equality can and even need to be addressed, given the wide impacts they potentially yield both temporally across generations and contemporaneously between the individuals that make up the society. All of this is, of course, one reason that such investments are public in the first place (e.g., Tuomala, 2009).

Public investments can be defined as one possible way of a government to promote economic growth and welfare in the society. Government in different countries undertake investments in both long-term projects like public infrastructure, and policy reforms, such as investments in R&D, health care or education. These investments are undertaken either as counter-cyclical policies, or to foster productivity and economic growth.

Given the second argument, that governments should utilise their large market shares, it can be asked, how large are the market shares of governments? Table 1.1 shows the %-share of GDP of *Public gross fixed capital formation* in Finland as well as a few comparison countries. Public, or, government gross fixed capital formation is usually what is meant, when talking about public investments. The numbers in table 1.1 are based on the System of National Accounts (SNA), which is a set of internationally standardised definitions and classifications for national accounting. The SNA's definition of public investments is at present defined as *Gross fixed capital formation* and *acquisitions, less disposals of non-produced non-financial assets* (OECD, 2015). Thus, both 'material' and 'immaterial' investments are included in the figures. The respective percentage figures for the Euro area vary on both sides of 3% for the years 2010-2014.

Another way of acquiring an idea of just how much is spent in public investments is to look at public investment's %-share of total government expenditures. According to OECD (2015) government investments in the OECD-area made up 7.8% of total government expenditures in 2013, though during the financial crisis that started in 2008, the level of public investments has been decreasing, because of the adoption of austerity programmes.

Perhaps an even more enlighting angle is to inspect ratio of public investments to total investments, since it was above mentioned that governments could use their relatively big market share. Table 1.2 shows public investments %-shares of total investment, which includes the investment spending of the entire economy. The public investment share of total investments seems to move between 10% and 20%, and the OECD average in 2013 of 15.9% (OECD, 2015) confirms this estimate.

Now, using this rough estimate of approximately 15% market share, it can be well thought, that the choices of government investments can have an impact on the markets. Thus, the quite generally acknowledged goal of aiming for a environmentally, socially and economically sustainable economy can be striven for, by valuing, for example, environmental issues in public investment decisions. Thus, public investment decisions matter.

Country\Year	2007	2009	2013
Finland	13.3%	18.7%	19.5%
Sweden	15.9%	20.5%	20.2%
UK	12.7%	22.3%	14.9%
USA	17.4%	24.2%	17.5%

Table 1.2: Public investment as a %-share of total investment, 2007, 2009, 2013 (Source: OECD.Stat)

In addition to public investments, the role of public procurement in steering private producers to sought after goals is worth mentioning. Public procurement can be defined to be a special case of public investment, which is strictly regulated by EU directives and national legislation. Public procurement and public investment are only partially overlapping categories, since even goods more akin to consumption goods can be procured. A considerable amount of money circulates in public procurement, as for example in Finland they amount to around 30 billion euros, or approximately 20% of GDP, annually. Thus, public procurement most definitely 'matters', and its use as a means of achieving goals such as sustainability has been the topic of both academic and practical discussions.[2] Further, where as public investments utilise Cost-Benefit Analysis, a similar method, Life-Cycle Costing has been increasingly taken up in public procurement decisions (see e.g., Gluch and Baumann, 2004; Pursimo, 2015).

The questions I try to answer in my thesis are the following:

- How should the social discount rate be chosen given different assumptions about the project under assessment? Particularly, what if the project's benefits are public goods and what if they are private goods?
- How is the social discount rate affected, if environmental externalities are accounted for? Can the externalities be thought of as a 'public bad' and acknowledged by adjusting the risk premium of the social discount rate?
- How do these theoretical suggestions compare to actual discount rate choices made by public authorities?

The use of a risk premium in social discounting seems like an enticing idea, given the difficulty of calculating certainty equivalents in a project assessment

[2]See, e.g., Alhola (2012), Lundberg et al. (2015).

context, and the Ramsey equation can be modified accordingly, introducing an expression for the social risk premium. The use of risk pooling, or diversification, would, however, seem intuitively logical in the context of public investments, and it precisely the idea of risk pooling that Arrow and Lind (1970) exploit in their seminal article, which proves the result, that, given certain assumptions, the social risk premium tends to zero as the size of the population grows large – a result known as the Arrow-Lind theorem.

It was later argued, that for the Arrow-Lind theorem to hold, the benefits and costs that the project yields need to be private goods (e.g., Fisher, 1973; Foldes and Rees, 1977). In the case of an environmental externality, which can be thought of as a 'public bad', the Arrow-Lind model's results need to be revised. Particularly, for the theorem to hold, the so called Hicks-Kaldor compensations from the beneficiaries of the project to the sufferers of the externality need to be actually carried out (Fisher, 1973).

Logically, a further question is to ask, what happens to the theorem's results given different degrees of rivalry, that is, degrees of 'privateness' of the projects benefits. The results should also be examined given different risk aversion profiles. In a more recent examination of the Arrow-Lind framework, Gallagher and Snow (2014) examine just these questions, and find that the original model's results do hold even with increasing rivalry of the benefits, given assumptions on the risk aversion profiles of the individuals. More precisely, the relative risk aversion and the measure of relative prudence need to be small enough for the theorem to hold.

The focus of this study is on the need of a social risk premium, given different assumptions. This is done by integrating the models of Arrow and Lind (1970), Fisher (1973) and Gallagher and Snow (2014).

This thesis is structured as follows. In chapter 2 I present a brief literature review into past research on the topic. Nex, in 3 I lay out the theoretical background of public investments and define public investments, social welfare functions, risk and uncertainty, and the theory of Cost-Benefit Analysis. To examine the need for a social risk premium in different situations, first in chapter 4 I formally derive the Ramsey equation, which defines the social discount rate, and second present a model in chapter 5 which integrates the models of Arrow and Lind (1970), Fisher (1973) and Gallagher and Snow (2014), to assess the need for a risk premium in social discounting. Finally, chapter 6 discusses and concludes.

Chapter 2

Literature

In this concise literature review I present relevant theoretical outings, first regarding Cost-Benefit Analysis, and then on the topic of Social Discount Rate.

Cost-Benefit Analysis (CBA) is a method for assessing the profitability of public projects. Alternatively, it can thought of as a method to be used in maximising societal utility (Marglin, 1969). According to Marglin, the ideas used in CBA can be traced back to Jules Dupuit in the 19th century, but as Marglin (1969) and Pearce (1983) point out, the idea was not put to practice until the Flood Control Act of 1936 in the USA. Since the 1930s, the use of CBA has spread widely and the theory, upon which it is based, has grown more cohesive, as is evident from the text-book type manuals on calculating it, published from the early 1970s onward.[1]

CBA, or other methods like it, has since then been used by many governments, particularly in the case of larger, or lengthier, projects, where the possible benefits, or losses, to the society are larger. However, as, for example, Rosen (2005) points out, even though USA has been a pioneer, when it comes to the use of these kind of present value methods in assessing public investments, they are actually seldom used even if the local law would require it. According to Rosen, the choice of a social discount rate for public project appraisals is also noncoherent within the US.

[1] Most notable are perhaps the works by Pearce (1983); Marglin (1969); Mishan (1994), all of which were iniatially published in late 1960s or early 1970s.

The use of CBA thus started before the formation of its welfare economical basis in the 1950s. These welfare economical ideas were the idea that benefits were any gains and costs any loss in utility, the use of the concept of opportunity cost, and rooting the idea of net benefit maximisation in the Pareto improvement rule. (Pearce, 1983).

A further note in the historical formation of CBA is that it emerged from a practical need, mostly in the context of water resource management (cf. The Flood Control Act). A relatively early Finnish text on the subject of CBA, Uimonen (1992), was also published in a book the main topic of which was water resource management.

In the 1970s CBA gained support from the nascent environmental movement. Though, it should be noted, the environmentalists were also among those, who most vocally criticised CBA for its expansion of economical reasoning to natural resources (Pearce, 1983). At present, the use of CBA is partly argumented for precisely because of the same reason, that is, acknowledging the environment. The inclusion of environmental factors in CBA calculations has indeed increased in recent decades, and has lead to a situation, where the acceptance of investments based on a CBA without them can be seen as questionable (Tuomala, 2009). The most obvious criticism towards CBA is aimed at it assigning monetary values to non-market goods. Taking this argument further, critics may claim that the use of CBA would overrule political judgement. Pearce (1983) addresses this strand of critique, and points out that CBA makes no claims of making morally correct decisions. Rather, Pearce writes, "[...] CBA is an 'input', an 'aid', an 'ingredient' of decision-making. It does not *supplant* political judgement". A good summary of why the environmentalists should be advocating the use of CBA, not objecting it is offered in Revesz and Livermore (2008).

Pearce (1983) further states that CBA is a 'normative' procedure and answers critiques of 'subjectivity' by saying, that the possibility that an analytical tool is biased by falsifications on behalf of the analyst is not something unique to CBA, and, not a part of the conceptual structure of CBA. CBA is, of course, 'subjective' in the sense that it does try to model the subjective preferences of the individuals in the society. Pearce suggests that not only should the value judgements that are made be made public, but even that the analysis should include a 'value judgement sensitivity analysis'. Such value judgements are presented in chapter 3, where I go through the theoretical foundations of CBA.

According to Pearce (1983) CBA could also be victim to the so called 'Scitovsky reversal paradox'. The paradox states that, given suitable preferences, it is possible that a project that takes society from point A to point B is an improvement according to CBA, but once there, the move back, from B to A, would also be an improvement, given the possibility of carrying out Hicks-Kaldor compensations from the beneficiaries of the project to the sufferers of it (Scitovsky, 1941).[2]

A Hicks-Kaldor improvement, a term based on the works of economists Nicholas Kaldor and John Hicks, is a weaker optimality condition than Pareto optimality, and it is defined as follows. Imagine a project yields benefits to a certain group of consumers, while another group suffers from it. Now, if there exists such a compensation that could be transferred from the beneficiaries to the sufferers, that would leave the sufferers as well of as before the project was undertaken, but still lead to an increase in the well being of the beneficiaries, then this project is called a Hicks-Kaldor improvement. It should be noted, however, that the Hicks-Kaldor compensations do not actually have to be carried out. The problem of intergenerational discounting could theoretically be overcome by the setting up of an 'intergenerational compensation fund' to actually carry out the Hicks-Kaldor compensations (Pearce, 1983). This same result is achieved by Fisher in his extension of the Arrow-Lind model (1973), and his model is used in part as a basis for the model presented in chapter 4.

Additional potential pitfalls include, according to Rosen (2005), the chain-reaction game, labour game and the double-counting game. The chain-reaction game is a symptom of a biased CBA, as it means that unnecessary secondary benefits are included in the calculations by the analyst to make the project seem more tempting, i.e. changes that are actually transfers are counted as benefits. The second 'game' is defined as the case when the wages of the workers employed by the project are read as benefits. The creation of jobs is, of course, a sound and common political goal, but it is not, however, a benefit in the calculation of CBA. The double-counting game is the case, when, for example, the selling price of a land area as well as the potential revenues of farming it were both

[2] Not to be confused with Scitovsky's later income-happiness paradox, presented in *The Joyless Economy* (1977).

counted, i.e. double-counting means that two benefits that are exclusionary are both counted.

CBA is highlightedly a method for public project appraisal, but its similarities to private investment decisions are, of course, many. Where as a private investor uses the market prices as a given, and uses them to make profit calculations, profit cannot usually be used as a decision rule for public investments (Tuomala, 2009). Although the primitive notion that the present and future are not equally valued is present both in private as well as public investment decisions, the choice of a discount rate is usually far more complex in public investments. The theory of social discounting is next discussed shortly, and it is then the focus of chapter 4.

Benefits and costs occuring in different periods are discounted in CBA, using a Social Discount Rate (SDR). The two fundamental arguments for societal discounting are:

1. social opportunity costs, and
2. a positive societal time-preference

(e.g., Marglin, 1969; Pearce, 1983; Boardman et al., 2014; Drupp et al., 2015). These arguments give rise to two candidates for the discount rate to be used in public investment decisions (Tuomala, 2009). The social opportunity cost is the certain return that the public investor could reap, if an alternative investment were made, and it is usually denoted with r. A positive time-preference means, that the individuals in the society value the present over and above the future, and it is often denoted with s, $1/\beta$ or δ, of which the last is adopted in the theoretical models I present in chapter 4.

Heal (2005) equates the social opportunity cost r with the social discount rate, which is intuitively correct, if r is written as a function of δ, among other things. r can thus be interpreted as either the social opportunity cost, or the equilibrium market interest rate defined by the preferences of the individuals in the society. It should be emphasised that this interpretation of r as a market interest rate is that of a equilibrium interest rate in a perfect economy. Hence, as Tuomala (2009) points out, if markets do not function flawlessly, then a public investor should not use the market rate for discounting.

In Pearce (1983), a simple two period model, the social time preference rate δ and the social opportunity cost r coincide only at the optimum, when the slopes of the production possibility curve and the social indifference curve are the same. In this situation, Pearce writes, it does not matter which rate we choose for CBA, but since r is far more easily observed than δ, we should use r. In practice the two interest rates seldom coincide. The existence of simple taxation of firms, for example, leads to r exceeding δ.

It should be noted, that δ often thought a value judgement, though not without limits as, for example, Anthoff et al. (2009) cite plausible values of 0-3. The time-preference rate is also dependent on other characteristics of the individual, and individuals from lower income percentiles often report higher time-preference rates (e.g., Lawrance, 1991). The accounting of different time-preference rates in an attempt to achieve increased equality is however, perhaps justifiably left out of consideration in practical implementations of CBA.

A further explanation for the difference between r and δ is, that individuals report differing discount rates in isolation and with the knowledge of a societal decision (Sen, 1967). It should, thus, be decided, whether a discount rate based solely on r or δ is adopted for CBA, or if an approach is adopted that tries to embody both the social opportunity cost and the social time-preference in the chosen discount rate. Heal (2005), for example, proposes the use of just the time-preference rate for long-run economy-wide projects. Usually, however, it is often logical to formulate a a combination of the two, by writing the discount rate r as a function of the time-preference δ as well as other variables, and use it instead. In chapter 4 the social discount rate, now denoted ρ, is thus derived using the so called Ramsey equation.

The Ramsey equation, based on the original model in the article by Frank Plumpton Ramsey (1928), has come to play the part of a benchmark model in discussions of social discounting, especially in the case of very long run projects.[3] The initial model by Ramsey was later extended by David Cass (1965) and Tjalling Koopmans (1965), to a model of economic growth where the savings decisions of individuals are endogenised. The model is, accordingly, often called the Ramsey-Cass-Koopmans model. The Ramsey equation states the equilibrium interest rate, which is to be used in social discounting, to be equal to the

[3]The archetypal very long run project is, of course, climate policy aimed at mitigating the impacts of climate change.

pure rate of time-preference plus the growth rate of consumption multiplied by the elasticity of marginal utility of consumption, the last term often labeled the 'wealth effect'.

In 2006 the economist sir Nicholas Stern, along with others, published a large report, in which the Ramsey equation is used, on the effects of global warming on the world economy, commissioned by the British government. The report is, as said, very large, but the part that economists most criticised was precisely the use of long-term societal discounting. Among the most notable critiques are those of Weitzman (2007), Nordhaus (2007) and Sterner and Persson (2008). The criticism is mainly focused on the Review's use of a near zero discount rate, which thus values relatively high the costs of climate change occuring far into the future. Among Finnish critical readings of the Review are those of Liski (2008) and Tuomala (2008). As Liski notes, the Stern Review can be seen as a massive CBA, and thus at least partially the same reasonings can be used to discuss the choice of a social discount rate for 'regular' public investments as well as for global climate policy.

In the case of very long and very large, even global, projects such as climate policy, CBA might prove ill-fitting. Instead large models of economic growth the PAGE model (Hope, 2006) used in the Stern Review could be considered. While Stern uses the PAGE model to dynamically model economic growth and environmental change, perhaps the most used model is the DICE model, developed by Nordhaus (1993). Nordhaus' model's basic setup assumes a pure intertemporal time-preference of 1,5% and the 'wealth effect' to be on the more conservative side of Hall's (1988) empirical estimates. Hall's empirical estimates for intertemporal substitution in consumption for this 'conservative side' are that a doubling of consumption leads to a marginal utility $1/4$ of original level. Liski (2008) notes, that in the Stern Review, a doubling of consumption leads to a decrease of only one half in the marginal utility. Finally, as Liski points out, Nordhaus' approach might not suit the context of the large uncertainties of climate change.

Heal (2005) notes, that problems arise in long term public decision-making if an infinite time horizon is adapted (as economists often do), if a real valued utility function is demanded, and if a utilitarian framework is adopted.[4] As should be

[4]Utilitarian means here that all generations should be treated equally.

clear by now, quite a lot of unanimity exists between economists on the correct social discount rate to choose. This discord is examplified by Heal's proposal to use the pure rate of time-preference, which he calls 'utility discount rate', when the choice is in a general equilibrium model, that is, when the investment to be made can have economy-wide effects. In partial equilibrium situations, Heal proposes the use of a social discount rate. The use of δ alone as discount rate in theories of public investment is quite unorthodox, and even more so in empirical cases, since δ cannot be easily observed and is thus almost certainly a value judgement.

The traditional Ramsey equation does not account for uncertainty, since it writes the social discount rate as a function of pure time-preference, the growth rate of consumption and the elasticity of marginal utility of consumption. Gollier (2001) extends the equation by introducing a rather complicated term which accounts for uncertainty. This added term by Gollier can be thought of as the social risk premium, which has been to topic of many economic articles. Most notably the aforementioned paper by Arrow and Lind (1970) builds a model on a state-preference framework by Hirshleifer (1965), in which the social risk premium vanishes given a large enough population, a result known as the Arrow-Lind theorem. The theorem has been the topic of quite a few theoretical papers, including the contemporaries Fisher (1973) and Foldes and Rees (1977), the first of which is used in chapter 4. More recent commentaries include those by Baumstark and Gollier (2014), Fesselmeyer et al. (2014) and Gallagher and Snow (2014), the last of which is also used in chapter 4.

It is worth mentioning that the original Arrow and Lind model and its critiques do not comment on the suitable time scale for the social discount rates they implicitly define. Thus, the results of the Arrow-Lind theorem, whether they hold or not, can be used to determine the need for a social risk premium in the discount rate suggested by the Ramsey equation, even in the case of more short-term investments.

Finally, it should be noted, that some economists have criticised social discounting itself as immoral. Among those who claim intergenerational discounting irrational are Ramsey (1928) and Pigou (1962). This criticism stems from the utilitarian idea of valuing each past, current and future generation's utility equally.

Chapter 3

Public Investments & Cost-Benefit Analysis

In this chapter I go through the theory of the basic building blocks of public investment decisions. First, investments and public investments are explicitly defined in 3.1 and in 3.2, respectively. I note that the definition of investment in the economic theory of decision-making might differ from ones used in, say, financial markets or national accounts. Second, the theory of Cost-Benefit Analysis is presented in 3.3. Finally, uncertainty and risk in the context of public investment decisions and social discounting are discussed in 3.4.

3.1 What are Investments?

The definition of an investment is to incur costs now and receive a benefit for it in the future. In other words, to invest is to forego present consumption or revenue in hope of added consumption or revenue in the future. This means that an economic agent, such as a consumer or a firm, sets aside some part of the disposable wealth that the agent could have used to increase her present utility and instead chooses to invest some of her wealth in an asset or project that leads to potentially increased possibilities of utility maximisation in the future.

The object of an investment can be either physical or intangible. This means that the aim of the public investment decisions can be to acquire anything from residential buildings to factories, highway bridges to railways and more abstract objects like computer systems or educational or health care policies. It can be defined, that the only thing necessarily in common between different investments, and the aspect that sets them apart from simple consumption, is the temporal dimension.

In the case of an individual consumer or a privately owned, profit maximising firm this definition of investment is rather straightforward. In economic theory firms use investments to maximise their revenue across time periods and individuals invest to maximise their utility across time periods. It is less clear what is the aim of a public agent, such as state, government or city, and what would lead to an increase in its utility. To address this problem, different versions of a societal utility function have been proposed in economic theory. What is of more interest in the context of this thesis, is how these investment decisions are made. Where as the optimal behavior of a private firm or a consumer is to choose the cheapest alternative that fulfils some minimum criteria, a public investor might use other decision criteria such as effects on the distribution of income. Generally, a public investor aims at acknowledging costs and benefits from the perspective of the entire society. Section 3.2 discusses questions of public economic agents as investors in more length.

Since investment decisions are decisions concerning the future, i.e. states of the world that have not yet come to pass, uncertainty and risk are always present to some degree. In their seminal book *Investment Under Uncertainty* Dixit and Pindyck (1994) describe three characteristics common to most investment decisions. These characteristics are:

1. the level of irreversibility,
2. the amount of uncertainty, and
3. the possibilities to change the timing of the investment.

Irreversible investment decisions have been of special interest for environmental economists. Large scale investments, such as dams, are for all intents and purposes virtually irreversible and have a potentially large impact on the surrounding environment. Considerations of irreversibility are also relevant in cases where the extinction of a species could result from the investment. See, for example, Arrow and Fisher (1974), Viscusi (1988), and Narain et al. (2007).

Uncertainty and risk in investment decisions can be formally characterised through the probability distribution of the possible outcomes of an investment project that is undertaken. Since the distribution is often unknown, *ex ante*, an estimate of it is attained, using expected values. Compiling these probability distribution requires expert knowledge and can be difficult, especially in the case of large, economy wide or even international projects. Once a satisfactory probability distribution of the possible costs and benefits is derived, a so called certainty equivalent should be calculated. Since in many cases the probabilities are not readily available and the calculation of cost and benefit specific certainty equivalents can be tedious work, an alternative approach is to adjust the used discount rate to account for risk, which is the topic of the model presented in 4. The following section on Cost-Benefit Analysis, 3.3, also discusses the problem of risk and uncertainty and defines them in a public investment context.

Investors then make their investment decisions taking risk into account to the best of their knowledge. One general rule that is often implicit in discussions on the topic is to use higher discount rates for riskier, more uncertain projects. This is because economic agents exhibiting risk aversive preferences demand a higher return to compensate for increased risk. The choice of discount rates and the derivation of the so called Ramsey equation, which defines a Social Discount Rate (SDR), are discussed in 4. Additionally, it can be thought, that the appraisal itsel of competing investment possibilities, i.e. the calculation of CBAs, reduces the uncertainty of the possible investments, as was proposed by Mirrlees and Little (2006).

The possibility of postponing an investment, and choosing the timing of the investment so that the uncertainty about the investment can be reduced by learning more before committing to the project, has been integrated into some of the decision aid computer programs that have been published in recent decades. These decision aid programs represent an alternative approach to providing public decision makers with theoretically based support in making investment decisions. These programs come from a combined background of engineering, economics and mathematics and are left out of consideration in this thesis.[1]

3.2 Public Investment Decisions in Economic Theory

As public investments fulfil all of the characteristics described in the previous section, what is it that sets them apart from private investments? What is the difference between a private and a public investment decision, besides the obvious fact that the decision maker is a public agent? It can be seen that the definition of public investment as offered by economic theory partially differs from the one used in practice, e.g. when a finance minister talks about the GDP-share of public investments, for instance.

When calculating national accounts following SNA, public investments are equated with the gross formation of fixed capital as defined in the first chapter of this thesis. These fixed assets can be both material and immaterial, and are used in production (of further middle products and/or end-products) for at least one year. This is the definition used in, for example, Statistics Finland. Public procurement can be seen as a special way of orchestrating public investment. Public procurement does not, however, fall completely into the category of public investment as defined above. This is because also end-products can be procured, where as the national accounts -definition of investment would only allow for differerent forms of capital as the object of investment. As noted in chapter 1, the use of CBA-like methods such as Life-Cycle Costing (LCC) in procurement has increased during the past decades, and these are discussed briefly in 3.3.1.

In economic theory any government action in or through the market mechanism is often called a market intervention. The market is thought of as functioning autonomously, so that any activity by the state is an inteference in the otherwise freely operating market. Different categorizations of the possible types of government intervention have been proposed. Pigou, for example, differentiates between two types of public expenditure: transfer and non-transfer (Pigou, 1947). Non-transfer expenditure means in Pigou's terminology public expenditure "that purchase current services of productive resources for the use of those

[1]See, e.g., Beuthe et al. (2000) andMedaglia et al. (2008).

authorities" where as transfer expenditure is that "which consist in payments made either gratuitously or in purchase of existing property rights to private persons".

More recent classifications include those by Joseph Stiglitz and Matti Tuomala. Tuomala categorizes all possible government market interventions as either direct or indirect. Direct interventions are defined as the government operating directly through the market mechanisms, and include public production, subsidies and investments. Indirect interventions are defined as altering the functioning of markets in some way, such as regulations and taxation. (Tuomala, 2009). Stiglitz's categorisation of different types of government market interference into three types is: public production, private production with taxes and subsidies and private production with government regulation aimed at ensuring that firms act in the desired way (Stiglitz, 2000).

Based on the above distinctions, it can be said that public investments are

- potentially a part of both non-transfer and transfer expenditure,
- direct interventions,
- either public production, or private production with subsidies.

Whether public investments fall into Stiglitz's category of public production or one of the two private production categories, is not altogether clear. An investment is essentially the government, or other public agent, buying something, for which the costs are paid immediately, or in the beginning of the investment period, and the benefits are reaped towards the end of the investment period.[2] It could be thought, that if the government invests in a product or service that is privately produced, then the said public investment falls into the category of private production with government subsidies. Similarly, in the case where government or some other public agent provides for the product or service that is the object of the investment, then the public investment falls into the category of public production.

According to Stiglitz (2000), the effects of a project should only be evaluated in the long-run. In other words, the impacts of a government market intervention

[2]This is a generalisation. It could of course also be the case, that a given public investment would require costs throughout its life-cycle, or yield benefits immediately.

should be analysed after all of the economic agents, who are stakeholders, have adjusted their behaviour accordingly. This approach presupposes the notion, that there exists some equilibrium state after which the consumers and firms do not alter their behaviour, until further actions (by the government) on the markets are made. This means that, although the idea is correct, in that it urges the decision-maker to take into account the full effect a project has on the behaviour of the economic agents, implementing it in practice might prove challenging, since it is difficult to draw a line in a point in time, when the affected economic agents have ceased altering their behaviour. This problem is in addition to the fundamental problem of economic theory of not having a laboratory setting, and hence not being able to consider the consequences of a project, *ceteris paribus*. This insight can, however, be used as a motivation for the use of CBA, which tries to assume a similar all-encompassing approach in assessing the costs and benefits of a given investment.

The reason why a government would want to intervene in the working of a market is, according to economic theory, market failure. Market failures can be divided into five different categories, namely: externalities, public goods, imperfect competition, incomplete markets and imperfect information (Stiglitz, 2000). It is quite clear, that all of these are present when evaluating the need for various public investments and even in the investment decisions of type 3, as defined below. How the proposed public investment decision-aid method, Cost-Benefit Analysis, answers the problems presented by the five types of market failures, is discussed in section 3.3.

After defining what public investments entail and how they fit into the public economic theory background, it needs to be defined just what is meant when talking about public investment decisions. I now introduce a distinction of three different types of decisions related to public investments. A public investment decision can fall into one of the following categories, which might overlap:

1. Decisions on the overall optimum level of market interference, through public investment, on a regional, local, national, or industry level
2. Decisions between entirely different public investment possibilities, e.g. new highway vs. educational reform,
3. Decision between alternative offers, when the decision number 2 has been made, i.e. if a highway needs to be built, should a section be done as

a bridge or a tunnel, or, which of the offers for a city-wide tram-system should be accepted

Cost-Benefit Analysis, an established set of theory in applied economics, is most often used to make decisions of the third kind, or to assess the profitability of a single investment option (e.g., Marglin, 1969). Thus, in this study, the main focus is on the third kind of investment decision. Although such a decision might seem mundane, that is, the alternative with the cheapest price tag should be chosen, in the case of public investments and the aim for maximising social welfare an alternative approach is needed. Economic journal articles around the topic, most notably the large body of research focused on discounting, take it as a given, that some form of Net Present Value -criterion, usually Cost-Benefit Analysis, is used. CBA is the topic of section 3.3.

Even though the definition of public investments I have formulated is rather broad, we now have a more clear picture of what it can mean. We should now turn to the question of how public investment decisions, of type 3., are made. From a theoretical perspective, the idea of maximising a societal welfare function as a criterion for investment decisions seems alluring, but, as we will see, is rather impractical. Subsection 3.2.1 briefly introduces social welfare functions and their short-comings, which mandates the use of a more crude method, CBA.

3.2.1 Social welfare & societal utility functions

The theory of social choice and welfare economics is, of course, not confined to the context of investment decisions. It can be generalised that a social choice problem is any situation whene a group of individuals are faced with making a collective choice from a set of alternatives (Jehle and Reny, 2011). The definition of a social welfare function is also related to Arrow's impossibility theorem and the search for an optimal social choice rule (Arrow, 1951).

A social welfare function is desired to fulfil at least some of Arrow's characteristics of a good social choice rule. The characteristics are the following. Unrestricted domain (U) means that the social welfare function is defined with all possible combinations of individual preference relations. Weak Pareto principle (WP) means, that if all individuals prefer x over y, then the rule should choose

x over y as well. Independence of irrelevant alternatives (IIA) is defined so, that the choice is the same irrespective of the presence of non-winning alternatives. Non-dictatorship (D) means, quite self evidently, that the preferences of a single individual should not over-rule the preferences of the other individuals. An additional feature of a good social choice rule is the so called Pareto indifference principle (PI), which states that if two alternatives are ranked equal by all individual preference relations, then the social welfare function should value them equally as well.

Jehle and Reny (2011) define social welfare functions as such, that satisfy U, WP, IIA and PI, i.e. the functions satisfy strict welfarism. The two most commonly cited forms of social welfare functions are of the Rawlsian and the Utilitarian form. The Rawlsian social welfare function, also called 'maximin', maximises the utility of the least well off individual in the society. The rationale behind this is derived from the ethical reasonings by Rawls (1988). The Utilitarian welfare function, which is perhaps more common in economics, is defined as the sum of individual utilities, and it ranks competing projects according to differences in this sum (Jehle and Reny, 2011).

From a theoretical point of view, the use of a certain social welfare function in public investment decisions seems justifiable. Rendering the theory to practice, however, can be quite difficult. First, the individual utility functions would need to be estimated. Second, the estimated utility functions would need to be such, that they would allow the adding up and comparison of utility levels of different individuals. Accordingly, as Tuomala (2009) points out, the different types of social welfare functions are mainly meant to be used in abstract reasoning, and are of little use in practical public investment decisions. Since a social welfare function is in practice very difficult to estimate, Cost-Benefit Analysis has been used as an alternative to such functions, in the context of public investment decisions, even though, according to Marglin (1969) it is only after the estimation of a societal utility function that "it is possible to make our notions of benefits and costs sufficiently meaningful that quantitative appraisal of economic alternatives within the public sector can be expected to lead to a higher level of performance than more intuitive methods of decision-making permit." Thus, the need for a more practical decision-aid method for social choices is well motivated. The proposed, and often used, method is Cost-Benefit Analysis, and it is to it that I turn next.

3.3 Cost-Benefit Analysis

I now introduce Cost-Benefit Analysis (CBA), following mainly Pearce (1983), Tuomala (2009), and Boardman et al. (2014). The basic idea of CBA is the following. Let us say the government wants to make a large scale investment, say a new highway, though CBA could just as well be used to assess more abstract public projects as well (Tuomala, 2009). The government then needs to have a way of deciding what is the economically and socially most efficient way of realising the highway. The socially optimal investment decision would maximise a social welfare function, but, as was noted in 3.2.1, in reality this is often not feasible. This is where CBA comes in. It is usually assumed, that CBA is used to make an economically rational choice, i.e. choosing an option where the gains from a project exceed the losses.

Analogous to the social welfare function, CBA aims at making choices according to the preferences of the individuals who make up the society. The market place and money are taken as the prime medium through which these preferences are expressed. This, however, leads to the implicit value judgement, that the current distribution of income in the society is preferable (Pearce, 1983). Pearce defines two normative judgements, which need to be made, in order to be able to use CBA in societal decision-making:

1. Individual preferences should count.
2. Those preferences should be weighted by the existing distribution of income.

Here, the latter of these normative judgements follows from the fact, that the market place represents the society's individuals' aggregated preferences, but biased according to the existing distribution of income. If judgement 2. is rejected, then an alternative needs to be presented, i.e. some other, desirable distribution of income needs to be defined in addition to a way of 'equalising' the current distribution, while using CBA. The former judgement is just a rephrasing of the fact, that CBA uses the market values of costs and benefits, which in turn are decided by the preferences of the individuals who act in it.

Since all of the benefits and costs of the project to individuals in the society should be included, the concept of consumer surplus is often adopted. In the simple case of a linear Marshallian demand curve, the area under the curve, in a quantity-price space of the representative single commodity, defines to consumer surplus. Even though more precise estimates of real demand curves could be estimated, Pearce (1983) points out that "[p]artly because of empirical difficulties, and partly because of the view that the errors are not large, we usually find CBA studies using 'simple' measures of surplus." Pearce defines three ways of calculating consumer surplus: (a) the area under a Marshallian curve, or 'simple consumer surplus', (b) the area under a Hicksian demand curve that is adjusted so as to keep the consumer on his original indifference curve, known as 'compensating variation', and (c) the area under a demand curve that is adjusted to keep the consumer on his subsequent indifference curve, known as the 'equivalent variation'.

According to Mishan (1994) compensated variation is the most often suggested approach in theoretical literature. A more recent text-book on CBA, by Boardman et al. (2014), presents the Marshallian demand curve for estimating the consumer surplus, as well as the Hicksian demand curve, and the equivalent variation versions. Thus, there is still no unanimity over which demand function to use.

Pearce's theoretical starting point for the net benefit criterion of project acceptance is the equation

$$Net\, benefits = WTP_i - WTP_j, \tag{3.3.1}$$

where the benefits of a given project i are presented as the willingness to pay for it, and the costs are represented as an opportunity cost, namely, the benefits (measured also through willingness to pay) of an alternative, foregone project j.

The above formulation is correct in a first best world, where markets are perfect, and prices thus perfectly capture the preferences of individuals. In reality, CBA calculations are done in a second best world. Then, WTP_j often differs from C_i, the costs of project i as measured in CBA. This means that the calculated costs fail to acknowledge the full societal costs. This mistake could be because of, say, externalities, or, the case when the inputs of project i are not valued equal to the values of their respective social marginal products.

When the market prices do not correctly reflect the societal preferences as expressed through marginal willingness to pay, the problem can be remedied by shadow pricing, i.e. trying to find the 'proper' measure of the opportinity cost of project i in terms of what the various inputs would produce in a first-best world (Pearce, 1983). According to Tuomala (2009), the shadow price can be thought of as a weighted average of demand and supply prices, that correctly depicts the societal marginal cost. The relative weights, of demand and supply prices, are defined as the proportion of added public demand (Tuomala, 2009). Shadow prices are thus needed precisely since they estimate the social marginal willingness to pay, which is what should be used in valuing benefits and costs in CBA, since we want to correctly measure net social benefits.

Pearce (1983) lists four qualifications that, even a 'timeless', CBA must fulfil. These are

1. The benefits and costs must be valued at shadow prices unless we have reason to believe that the error involved in not using shadow prices is small.
2. All costs and benefits, to whoever they may accrue, must be accounted for.
3. The rule is indifferent between who receives the benefits and who suffers the costs.
4. A positive net benefit does not mean that the project should be undertaken.

These rules can be justified followingly. Shadow prices, instead of market prices (for those goods for which these exist), must be used, in order to capture the full societal opportunity cost involved. And, following similar reasoning, all costs and benefits must be accounted for, or, in other words, both internal and external costs must be accounted for. This means that all possible externalities, no matter how miniscule, should be assigned a monetary value. This second ruling is perhaps the most troublesome, when applying CBA in practice. The third rule just states that distributional considerations are not included in this simple CBA formulation, and the final rule just rephrases the theoretical acknowledgement, that CBA alone is not and should not be used as a decision-rule. Underlining the need for additional decision-aid methods besides CBA, Tuomala

(2009) writes, that practical implementation requires case-specific deliberation and thought, as well as plenty of 'common sense'.

We can now rewrite (3.3.1) as follows. If C denotes a sum of all the costs, correctly reflecting the social opportunity costs, related to a project and B denotes the benefits, we then have the following 'timeless' criterion for the investment to be undertaken:

$$PV = B - C > 0, \tag{3.3.2}$$

i.e. the net benefits must be positive, for the project to be worthwhile.

This is, of course, a confusingly simple idea, but the implementation of it is far from straightforward. Problems arise when trying to monetise 'non-market' benefits or costs, and by introducing the temporal dimension into the calculation, things get even more difficult. Another possible cause of difficulties for calculating a CBA is defining boundaries, i.e. which costs and benefits to include.

Costs and benefits occuring at different points in time should, of course, be made commensurable by discounting them to the value of a certain point in time. The need for this discounting can be argumented in a multitude of ways. The most usual arguments (e.g., Boardman et al. (2014); Pearce (1983, 1999)) are: the social opportunity costs, and, the actual time-preferences of consumers, who are usually assumed to be impatient, or myopic. This leads to two candidates for a social discount rate, the pure rate of time-preference δ, and the market interest rate r. Depending on the particular context, either δ, r or a combination of them, noted ρ, is used. A particular rule called the Ramsey equation, which states ρ as a function of δ and other variables, is formally derived in chapter 4. For now, I will use ρ. Additionally, it should be noted, that the discount rate does not necessarily have anything to do with inflation, since the rationale behind individual and social discounting is independent of any expectations of future changes in the price levels.

Now, since the first preliminary 'value judgement' stated that consumer preferences matter, a zero discount rate approach cannot be defended in our setting. From a perspective of intergenerational equality, the use of a zero discount rate has, however, been defended by, for example, Ramsey (1928) and Pigou (1947), and more recently in the context of climate change (Davidson, 2014).

The net value criterion (3.3.2) can now be written as the net present value criterion

$$NPV = \sum_{t=0}^{T} \frac{B_t - C_t}{(1+\rho)^t} > 0, \tag{3.3.3}$$

or, in the case of a continuous time model, when it is customary to use exponential discounting, we have

$$\int_0^T \left(B_t e^{-\rho t} - C_t e^{-\rho t}\right) > 0. \tag{3.3.4}$$

Additionally, it should be noted, that different rules for choosing a social discount rate should be used based on the length of the project, the effects on the general equilibrium (i.e. local vs. economy-wide), and the degree of rivalry of the costs and benefits of the project (i.e. public goods vs. private goods).

In the case of non-market goods, Pearce (1983, 1999) distinguishes between two possible ways of deriving a monetary value for them. First, the value of the non-market good might be present in the prices of a surrogate market (e.g., the value of peace and quiet can be thought to be expressed in the prices of houses). Second, the value of non-market goods with no surrogate markets can be found out by inventing a market, usually by asking people what their willingness to pay for the good in this hypothetical or experimental market would be. These approaches are theoretically sound, but difficult to implement in practice (Pearce, 1999). Particularly difficult costs to estimate include the 'option', 'existence', and 'bequest' value Pearce (1983).[3]

Since CBA is used with the aim of a socially optimal solution, the concept of Pareto optimality, a situation where no agent could be made better of without lessening the utility of another agent, needs to be accounted for. In a first-best world, where prices correctly reflect social preferences, CBA leads to Pareto optimal decisions. One practical way of checking this is to use the so called Hicks-Kaldor potential improvement test, as proposed by, for example, Boardman et al. (2014).

[3]Here bequest value means the value of being able to hand something down to future generations, existence value is the intrinsic value of something without the option of using it, and option value denotes the value that individuals place on the option of being able to use a certain item, e.g., natural area.

Whereas conventional CBA implicitly assumes that the existing distribution of income in the society is optimal, by assigning equal weights on all, i.e. value judgement 2. is made, there exists an alternative approach, the advocates of which are called 'revisionists' by Mishan (1982). Pearce (1983) denotes a further distinction of two differing approaches within the 'revisionists'. The first is to derive a single set of weights by looking at a political decision-maker's objectives. Marglin (1969) and Mirrlees and Little (2006) can be seen as examples of this approach, and it is used mainly in the context of developing countries. The second approach is to let the analyst perform a sensitivity analysis of the CBA's results by trying multiple sets of distributional weights.

Pearce (1983) answers many criticisms, mainly those of Mishan (1974; 1981; 1982), of using distributional weights in CBA.. The first line of criticism, that some assigned weights might lead to an even steeper inequality of distribution, is invalid in that it does not acknowledge the distributionally weighted CBA's aim of incorporating aspects of both efficiency and equality. The second argument, that the weights are arbitrary does not hold, since either they are referenced from explicit objectives of politicians or, in the 'sensitivity analyst approach', they are many and the point exactly is to compare the results of different, arbitrary, weights. Finally, Mishan's critique, that the analysis loses its economic objectivity since the weights might change with the change of a political regime, is besides the point since for the 'sensitivity analysts' the same argument holds as for the second criticism, and for the single-set weighting the context is often that of a developing, centrally planned country, in which the weights assigned by the leading politicians are precisely the ones that matter, from the planner's point of view.

In reality, a non-weighted version of CBA is often used. This is mainly because deriving of such weights is laborous and difficult, and often there is not a single clearly correct way of deriving them. (Boardman et al., 2014). Tuomala (2009) notes, that even though distributional issues gain attention in public discussions, they have usually been averted in CBA, perhaps based on the musgravian notion of separating the functions of resource allocation and income redistribution (Musgrave, 1959). Boardman et al. (2014) suggest limiting the use of a weighted CBA to only those projects, whose primary goal is to improve the standing of some particular group, i.e. projects in which distributional considerations play a central role.

To conclude, it should be noted, that there are competing and similar methods to CBA. These include calculating a benefit-cost ratio and using the so called internal rate of return. Benefit-cost ratio as a method is quite self-explanatory. The internal rate of return is defined as the discount rate, which would make the present value of a project's net benefits equal to zero (Rosen, 2005). Using the internal rate of return, the project if chosen, if the internal rate of return exceeds a chosen comparison discount rate (e.g., the market rate).

In the presence of a budgetary constraint, when the task is not to choose a single optimal project, but the best combination of projects available with the given budget, Pearce (1983) suggests using benefit-cost ratio instead of CBA, since CBA might lead to socially non-optimal solutions. Contrary to this, in the case when choosing just one optimal project, CBA leads to better decisions than benefit-cost ratio or rate of internal return, since the alternative methods can lead to erroneous choices, for example because of a difference in the scale of compared projects (Rosen, 2005). According to Tuomala (2009) the internal rate of return is almost impossible to use in the presence of time-variant interest rates, i.e. when the assumption of a constant rate is relaxed.

3.3.1 Life-Cycle Costing

Life-Cycle Costing (LCC) is a method quite similar to CBA. The main difference between the two is that by default only costs are taken into consideration when using LCC, where as CBA calculates the difference between the costs and benefits of a given project, as explained in the last subsection. This means that, where as CBA suggests going through with a project only if its net present value exceeds zero, LCC suggests choosing a project whose total costs are the lowest, i.e. closest to, but still below, zero.

Thus the criterion is

$$LCC_i = \sum_{t=0}^{T} \frac{C_t}{(1+\rho)^t}, \tag{3.3.5}$$

and the alternative i, for which the expression in (3.3.5) is the lowest, should be undertaken.

The term LCC is used usually in the context of public procurement, given that the EU directives and national legislations on public procurement directs procurement officials to using it in tender evaluation. Public procurement can be seen as a specific type of public investment as defined in 3.2. Given the fact, that procurement usually deals with smaller and shorter investments, the usual exclusion of externalities in LCC calculations is understandable. In recent years, however, the use of environmentally extended LCC tools has gained increased attention (see, e.g., Sterner (2002); Hunkeler et al. (2008)). A survey of current LCC tools used in public procurement can be found in Pursimo (2015). A good discussion of the role of LCC in environmental decisions-making is presented in Gluch and Baumann (2004).

3.4 Risk and Uncertainty

Following Pearce (1983), I define risk and uncertainty separately. Risk can be characterised as the probability distribution of the possible outcomes, when the distribution is known *a priori*. Uncertainty is defined as the uncertainty over the probabilities of the possible outcomes in the case when the probability distribution is not known. In the latter case, coming up with simple decision-rules is more difficult than in the former situation (Tuomala, 2009).

Different solutions to the problem of uncertainty, as defined above, stem from the branch of economic theory called decision theory (Pearce, 1983). According to Tuomala (2009), one possibility would be to use expected values, assigning to them either objective, based on previous experience, or subjective, based on expert recommendations, probabilities.

There is no sure-fire, correct way of deriving a plausible probability distribution for the possible costs and benefits. Four possible rules for deriving the distribution are: maximax, maximin, index of pessimism and minimax regret. Maximax suggests using the highest possible value of each cost and benefit. Minimax suggests, in a Rawlsian fashion, using the worst possible outcome in each case. When using the index of pessimism, a subjective probability weighting is given to the best and worst possible outcome for each cost and benefit, and then this weighted average is used. Finally, 'minimax regret' is used by assigning 'regret' values to all possible values of costs and benefits as the difference between the best outcome and the given value. In this way, a 'regret matrix' is formulated, from where the largest 'regret' values for each cost category are then chosen. (Pearce, 1983).

The problem with using simple expected value is, that it does not account for the level of risk aversity that the preferences of society, or individuals, exhibit. A way around this is to use expected utility instead of expected value. Using expected utility, a so called certainty equivalent is calculated, which is used to calculate the present values of the assessed project. Deriving the certainty equivalents of costs and benefits would require knowledge of society's utility function with respect to the costs and benefits in question. There is, however, no certain or easy way of observing these utility functions, given that we live in a second-best world (e.g., Pearce, 1983).

An alternative to deriving probability distributions, using them to calculate CBAs and then conducting sensitivity analyses, is the use of a discount rate adjusted for risk. The calculation of sensitivity analyses is strenuous, and furthermore, out of the scope of this thesis. The use of the risk premia, on the other hand, can be further motivated by stating, that risk and uncertainty are often functions of time, so that the farther a benefit or function is in the future the less certainty there exists over its value. A risk-premia adjusted, higher, discount rate takes precisely this into account by assigning lower values to those benefits and costs that occur far into the future.

The introduction of a social risk premium into the formula which gives us the equilibrium social discount rate, is precisely what Gollier's 'precautionary saving' signifies (2001). The need for a risk premium in public investment decisions has been challenged, most notably by Arrow and Lind (1970), whose model is used as a basis for the model I present in section 5. If the Arrow-Lind theorem should hold, and no adjustment for risk in the social discount rate were needed, then the calculation of probabilistic costs and benefits in a CBA would be easier. As critics, including Pearce (1983) and Tuomala (2009), have pointed out, however, the theorem does not seem to hold in a majority of situations. Two critical extensions of the Arrow-Lind framework are presented in chapter 4.

An important further notion by Pearce (1983) is that in the case of uncertain future costs, we would like to increase their values in CBA, but a (positive) risk premium does the opposite, since the larger the discount rate, the smaller the value of costs occuring in the future. Pearce's criticism is not valid, since, in the case of uncertaint of future costs, the risk premium could just as well receive negative values, and hence lower the discount rate and increase the valuation of future events. Just like the choice of the social discount rate itself is dependent on the size, reach and length of the proposed project, so is the justification of the Arrow-Lind theorem and hence the use of a social risk premium. A rule of thumb could be, that if the project is not large and has no large effects to the rest of the economy, then the costs of risk could be omitted.

As I have shown in this chapter, economic theory has many competing answers to the problems posed by uncertainty, risk and discounting in a public investment context. It should be further noted, that the risk in this discussion is usually a microeconomic risk, which means that natural disasters, wars, or other such economy-wide 'catastrophical', macroeconomic risks are not accounted for.[4] These answers are, however, markedly theoretical, and hence actual cost-benefit calculations can be expected to use simpler and cruder procedures. Given the appeal of conducting a risk free CBA and using a riskless social discount rate, it is to that specific question that I turn to next, as chapter 4 analyses the need for social risk premia in a variety of situations.

[4]However, it has been known, that a separate 'catastrophic risk' variable is incorporated in the risk premium of a Ramsey equation in practical implementations, as is done, e.g., in the guidelines by the Swedish Transport Agency (Trafikverket, 2015).

Chapter 4

Social Discount Rate

Investment decisions are decisions on costs and benefits occuring at different points in time, as was defined in chapter 3. These costs and benefits are made commensurable by discounting them to the value of a certain point in time, usually the time of purchase, using the Net Present Value -methods (NPV), that are of the form

$$NPV = \sum_{t=0}^{T} \frac{B_t - C_t}{(1+\rho)^t},$$

where the coefficient $\frac{1}{(1+\rho)}$ is called the discount factor, ρ denotes the real discount rate, and the exponent t denotes the period in the future from which the cost, or benefit, is discounted. To be able to calculate the NPV of a given project, the investor must choose the discount rate, ρ. The discount rate used to assess the costs and benefits of a public investment is often called social or societal discount rate (see, for example Arrow and Lind, 1970; Marglin, 1969; Mendelsohn, 1981; Stiglitz, 2000; Tuomala, 2009).[1] This term puts emphasis on the fact that the discount rate used by a public investor might or even should differ from that used by a private investor.

There is, however, no universal, generally agreed upon method of choosing the discount rate to assess a given investment, as discussed in chapters 1 and 2. This ambiguity is true for private investors and even more so when choosing a

[1] Sometimes abbreviated to SDR (e.g., Boardman et al. (2014)).

discount rate for a public investment, as is evident from the large amount of academic discussion devoted to it. Even among the experts of social discounting, opinions on the correct level of discount rate, and on the correct method or rule for choosing it for a public project vary considerably (Weitzman, 2001; Drupp et al., 2015).[2]

The need for a social discount rate different from zero stems from the fact that the same nominal amount of money, or utility, is worth different amounts to an investor in different points in time. The intuition is that assets available in the future are worth less than the same amount available today, from today's perspective. This is traditionally thought to be because of two reasons, namely, opportunity cost and impatience or myopic preferences (e.g., Boardman et al., 2014; Pearce, 1983), as noted in section 3.3 as well.

The first argument, opportunity cost, is as follows. If the market interest rate is r then 1 € invested this year is worth $(1 + r) \cdot 1$€ next year. If we then ask the question how much is one € next year worth to us from this year's perspective, then the answer must be $\frac{1}{1+r}$€. This is, because, if we invested this amount today, it would be worth exactly one € next year, i.e. $\frac{1}{1+r} \cdot (1+r) = 1$. Following similar reasoning, the value of € two years into the future is $\frac{1}{(1+r)^2}$, and $\frac{1}{(1+r)^t}$ for t years into the future.

The discount rate used for public investment decisions should then represent the societal opportunity cost, the returns r on alternative investments. The societal opportinity cost can be thought of as the private, and public, economic activities that would be crowded out, i.e. not undertaken and partly replaced, if the public project under question would be carried out (Tuomala, 2009).

The second argument, impatience or myopia, short-sightedness, enters into the derivation of a discount rate when the discount rate is defined as being made up of certain elements, one of which is called the social time-preference, i.e. a measure of the impatience in the society, denoted with δ. One of the first formal presentations of the social discount rate, and one that, despite being formulated a long time ago, is still used in discussions on the topic, is that of Ramsey

[2]Since social discounting is not a discipline in itself, the moniker "expert" in this context should be taken with a grain of salt. In their recent working paper Drupp et al. (2015) present evidence from a survey on 197 academics that they define as "experts" of long term social discounting.

(1928). The result, later coined the Ramsey equation, is usually defined (e.g., Drupp et al., 2015) as

$$\rho = SDR = \delta + \gamma g, \tag{4.0.1}$$

where δ is the pure rate of time-preference, γ denotes the elasticity of marginal utility of consumption and g is the growth rate of per-capita consumption. The interpretation of δ is that for an individual who is indifferent between receiving a benefit today or in the future, the value of δ would be equal to unity, where as a value for δ in excess of unity depicts the preferences of an impatient individual.[3]

An alternative taxonomy of reasons for discounting is offered by Gollier (2001), who defines three arguments, namely, pure rate of time-preference (δ), the 'wealth effect' ($\gamma \cdot g$), and precautionary saving. The wealth effect is defined as the effect of the presence of expected economic growth g, and thus enhanced future consumption possibilities, on consumers' saving behaviour.[4] The final argument, precautionary saving is a measure of the consumers' reaction to uncertainty regarding the future, and it is not found in the simple formulation of the Ramsey equation in (4.0.1). Gollier's approach is used in section 4.1 to formally derive the Ramsey equation, which defines the equilibrium discount rate as a function of the three aforementioned arguments.

The rest of this chapter is dedicated to formally deriving the so called Ramsey equation following Gollier (2001), Liski (2008) and Palokangas[5]. The Ramsey equation, which is based on the original writings of Ramsey (1928), is still used in academia as well as actual investment decisions some 90 years after its presentation. For example, the Swedish state organisation for transportation uses it in its internal guides to motivate their choice of SDR (Trafikverket, 2015), and it is still used as a benchmark framework in theoretical economic articles on the topic, as for example in Davidson (2014), Ahlvik (2015) and Drupp et al. (2015).

[3] The possibility that $\delta < 1$, i.e. that the individual would value future consumption above current consumption is of course also possible but is left out of consideration here.

[4] I.e. individuals value present consumption possibilities above future consumption possibilities since they expect to be better off in the future anyway.

[5] Tapio Palokangas, personal communication, 31.3.2016.

4.1 Ramsey Equation

I will now derive the so called Ramsey equation following mainly Gollier (2001). First, a risk-free version of the rule is defined, which is then expanded in the case of uncertainty regarding the future. The need for an expression of uncertainty in the equation, i.e. a risk premium in SDR, is then examined under different conditions in sections 5, 5.2 and 5.3.

The economy consists of identical consumers, who maximise their time-separable utility function u by consuming the single commodity c given the limitation of their disposable income A, which is an exogenous random variable that grows at the rate g.[6] The commodity is assumed such, that it cannot be saved over periods, $t = 0, \dots T$. However, the consumers can borrow and lend the commodity over time. The equilibrium exchange rate for c_{t+1} against c_t is the equilibrium price of time.

Next, $v_t(A, b)$ is defined as the value function at period t of a consumer who has the disposable income A and has to pay back the amount b of his debt. The consumer thus maximises

$$v_t\left(A, b\right) = \max_{c_t} \left\{ u\left(c_t\right) + \delta^{-1} E v_{t+1}\left(A\left(1 + g_{t+1}\right), \rho_t\left(c_t + b - A\right)\right) \right\}, \qquad (4.1.1)$$

with $v_T\left(A, b\right) = u\left(A - b\right)$, i.e. all debts have to be paid off in the final period. At date t the agent gets the income A, consumes c_t of the commodity, pays back the debt b, and then borrows the difference $c_t + b - A$ at the interest rate ρ_t. In the following period, the consumer's disposable income will be $A\left(1 + g_{t+1}\right)$, and he will now have to pay back $\rho_t\left(c_t + b - A\right)$. Differentiating (4.1.1) and equating with zero yields the following first order condition for optimality.

$$u'\left(c_t\right) = -\delta^{-1} \rho_t E v'_{t+1}\left(A\left(1 + g_{t+1}\right), \rho_t\left(c_t + b - A\right)\right) \qquad (4.1.2)$$

where $v'_{t+1} = dv_{t+1}/db$. According to Gollier (2001) in this simple model, autharchy is a competitive equilibrium. Thus, the utility maximising consumers do not borrow nor lend at any date t. This leads to them consuming $c = A - b$

[6] Since A can be thought of as GDP per capita, its growth rate g can approximated with the growth rate of GDP. For example, Liski (2008) discusses using the growth rate of the economy as a proxy for the growth rate of per capita consumption, which is needed in the Ramsey equation.

in each period, where b is the amount that they will reimburse of any past loans. Using this, we can solve an expression for $\frac{dv_t}{db}$ from (4.1.1) as

$$\frac{dv_t}{db} = u'(c) = -u'(A-b)\,. \tag{4.1.3}$$

Since there occurs no lending, we have $c = A$, which means that g_{t+1} now denotes the growth rate of consumption, and we define $c_{t+1} = (1+g_{t+1})\,c_t$. We can now solve the equilibrium interest rate from the economy's Euler equation, derived formally in appendix A.1, which is of the form

$$\frac{1+\delta}{1+\rho} = \frac{u'(c_{t+1})}{u'(c_t)}. \tag{4.1.4}$$

The expression for ρ, the equilibrium interest rate, which should be used for discounting, thus becomes

$$\rho_t(c_t) = \left[\frac{u'(c_{t+1})}{u'(c_t)} \cdot \frac{1}{1+\delta}\right]^{-1} - 1$$

$$\rho_t(c_t) = \frac{u'(c_t)}{u'(c_{t+1})}(1+\delta) - 1. \tag{4.1.5}$$

The rationale behind discounting is often argumented with two reasons: impatience and opportunity cost (e.g. Boardman et al. (2014)) as discussed in section 3.3. Gollier, however, defines three separate reasons to adjust the discount rate, namely, impatience, the 'wealth effect', and precautionary saving.

To consider the impact of impatience on the determination of the discount rate ρ, let us assume that the growth rate g equals zero. Then we have from equation (4.1.5) the result, that $\rho = \delta$, i.e. the discount rate equals the pure rate of time-preference. Usually individuals are assumed to be impatient, i.e. value the present over the future, and hence $\delta > 1$.

Following Gollier (2001) a certain growth rate of the economy is now introduced, to examine the effects of willingness to smooth consumption over time on the discount rate. The presence of positive economic growth means that the discount rate ρ no longer equals the pure rate of time-preference, since consumers expect larger revenues in the future due to economic growth, and hence want to borrow today against the future. The interest rate is now defined by equation

(4.1.5), where, since u is concave and $g_{t+1} > 0$, then $u'(c_t) > u'(c_t(1+g_{t+1}))$ and thus $\frac{u'(c_t)}{u'(c_t(1+g_{t+1}))} > 1$. Then (4.1.5) is larger than δ. This verifies the intuitive notion, that the discount rate should be larger in the presence of positive economic growth than without it, for the reasons explained above.

By looking at the expression for ρ in (4.1.5) we can see, that an increase in the concavity of the consumer's utility function will lead to increased willingness to smooth consumption over time. We can formulate the following proposition.

Proposition 1. *If there is no uncertainty about the future rate of economic growth, then an increase in the concavity of the consumer's utility function leads to: (1) an increase in the equilibrium risk-free interest rate, if the growth rate is positive, (2) no change in the equilibrium risk-free interest rate, if there is no economic growth, (3) a reduction in the equilibrium risk-free interest rate, if the growth rate is negative.*

Now, supposing that the growth rate g_{t+1} is very small, we can take a first order Taylor approximation from (4.1.5) around c_t. The Taylor approximation is formally derived in Appendix A.2. We get

$$\rho_t(c_t) \approx \delta + g_{t+1}\gamma(c_t), \tag{4.1.6}$$

where the term $\gamma(c_t) = -c_t u''(c_t)/u'(c_t)$ is the elasticity of marginal utility of consumption, or fluctuation aversion as Gollier calls it, or, a measure of the concavity of u. Still another interpretation for γ is, of course, the Arrow-Pratt measure of relative risk aversion (e.g., Kimball (1990)). Equation (4.1.6) is the no-uncertainty version of the Ramsey equation, and it tells us that the social discount rate is equal to the pure rate of time-preference plus the elasticity of marginal utility of consumption multiplied with the growth rate of per capita consumption. Thus the effect of a certain non-zero growth rate on the discount rate is approximated by $g_{t+1}\gamma(c_t)$. The idea, that consumers want to value present consumption more and discount future consumption more heavily, based on knowledge of a positive growth rate, is called the wealth effect (Gollier, 2001; Liski, 2008).

To relax the unrealistic assumption of certainty on the future growth rate, Gollier introduces the 'precautionary equivalent' growth rate, following Kimball (1990). The precautionary equivalent growth rate is the certain growth rate that leads to the same interest rate, that would be reached under uncertainty on the growth rate. The precautionary equivalent growth rate, $\hat{g}_{t+1}$, is thus defined implicitly in

$$Eu'\left(c_t\left(1+g_{t+1}\right)\right)=u'\left(c_t\left(1+\hat{g}_{t+1}\right)\right). \tag{4.1.7}$$

Gollier then writes out an approximation of the precautionary equivalent growth rate, which is analogous to Kimball's (1990) approximation of a precautionary equivalent risk premium, as

$$\hat{g}_{t+1}\approx Eg_{t+1}-\frac{1}{2}\sigma^2_{g_{t+1}}\frac{-c_tu'''(c_t)}{u''(c_t)}. \tag{4.1.8}$$

Here, $\sigma^2_{g_{t+1}}$ is the variance of the growth rate and $P=-c_tu'''(c_t)/u''(c_t)$ is the index of relative prudence, i.e. a measure of how much the consumer will adjust his savings as a reaction of increased uncertainty about the future. Equation (4.1.8) states, that the effect of uncertainty on the growth rate of consumption is equal to a certain decrease of the growth rate by half its variance multiplied by the consumer's relative prudence. The more prudent the consumers, the larger the decrease in the discount rate as a reaction to it. A necessary condition for this is, that the first derivative of the utility function, u', be convex. Hence, P can also be interpreted as a measure of the convexity of u' (Liski, 2008).

Given Proposition 1, we know that, if $g_{t+1}=0$, we have $\rho=\delta$. Using this, we can formulate the following proposition.

Proposition 2. *If the precautionary equivalent growth rate is zero, $\hat{g}_{t+1}=0$, then an increase in the degree of prudence, P, leads to a reduction in the risk-free rate ρ.*

To extend the reasoning of Proposition 2 to the situation when the precautionary equivalent growth rate gets other values, and to arrive at a formulation of the Ramsey equation with uncertainty of future economic growth, (4.1.8) is plugged in (4.1.6) replacing g_{t+1}. We get

$$\rho_t\left(c_t\right)\approx\delta+\left[Eg_{t+1}-\sigma^2_{g_{t+1}}\frac{-c_tu'''\left(c_t\right)}{u''\left(c_t\right)}\right]\frac{-c_tu''\left(c_t\right)}{u'\left(c_t\right)}$$

$$\rho_t\left(c_t\right) \approx \delta + E g_{t+1} \gamma\left(c_t\right) - \frac{1}{2} \gamma\left(c_t\right) P\left(c_t\right) \operatorname{Var}\left(g_{t+1}\right). \tag{4.1.9}$$

This expression of the Ramsey equation denotes the equilibrium discount rate as a function of all three of Gollier's arguments, since in (4.1.9) δ is the rate of pure time-preference, $E g_{t+1} \gamma\left(c_t\right)$ is the 'wealth effect', and $-\frac{1}{2} \gamma\left(c_t\right) P\left(c_t\right) \operatorname{Var}\left(g_{t+1}\right)$ denotes the effects of precautionary saving. Liski (2008) reaches the precisely same formulation for the Ramsey equation, using an even simpler two period model using exponential discounting. Gollier (2001) notes, that a similar formulation was reached by Hansen and Singleton (1983) using a continuous time model, and that the approximation (4.1.6) is exact in the continuous-time limit.

We now have an expression for the effect of uncertainty on the equilibrium discount rate, which can be interpreted as a sort of risk premium. An intuitively appealing idea, that public investments could omit this risk premium from their social discount rate, due to the possiblity of risk pooling, is next formalised by introducing the Arrow-Lind theorem (Arrow and Lind, 1970) in chapter 5. Given the rather stringent assumptions needed for the theorem to hold, the Arrow-Lind model is then extended further in 5.2 and 5.3, to see how the need for a societal risk premium varies given different assumptions about the economy.

Chapter 5

The Model

In this chapter I first derive the basic model to determine the need for a societal risk premium in the SDR, following the seminal article *Uncertainty and the Evaluation of Public Investment Decisions by* Arrow and Lind (1970). I then integrate into the basic model two extensions, the introduction of an environmental externality in 5.2, based on Fisher (1973), and, finally, the effects of differing degrees of rivalry in the benefits and costs of the public investment given different risk aversion profiles in 5.3 following Gallagher and Snow (2014). The model is used to prove, that given certain assumptions, the risk premium of each individual consumer and the total social risk premium goes to zero, as the size of the population gets large enough.

5.1 Basic Model

Using a willingness-to-pay approach and examining public investment as an investment in which each consumer, or tax payer, has a very small share, expressions for risk premia are formulated as follows. In this simple setting the temporal dimension is abstracted away, since the effects of risk aversion are to be examined separately from time-preferences. The results of the first half of Arrow and Lind (1970), wherein the same conclusion is reached that, given certain assumptions, the discount rate to be used in public investments should be one suitable for certain returns, are presented in appendix A.3.

The economy is comprised of n identical consumers, whose disposable incomes are represented by the identically distributed random variable A plus a mean-zero random element ε. However, since ε has no effect on the results which are derived, it can be abstracted away for a more parsimonious model. The consumers aim to maximise their utility, denoted $u(A)$, a continuous strictly increasing von Neumann-Morgenstern utility function. Consumers are assumed to have identical prefences, which exhibit risk aversion. Further assumptions about the type of risk aversion need not be made, for now.

θ denotes the returns of a government investment, which are independent of A. θ is further divided into a certain part and a random, mean zero risk element, denoted $\bar{\theta} = E(\theta)$ and $X = \theta - \bar{\theta}$, respectively. As θ is independent of A, so is X.

An individual consumer's share of the government's project is represented by s, $0 \leq s \leq 1$. The individual's disposable income can then be written as $A + s\theta = A + s\bar{\theta} + sX$. For simplicity, each consumer is assumed to have the same share of the returns. Thus, we have $s = 1/n$. s can additionally be thought of as each individual's 'tax rate', along the following reasoning. Given that the consumers exhibit risk aversion, they value their respective benefits from the project below the expected value, $\bar{\theta}$. If the random element of the project's returns, X, yields positive returns, this is lessens the tax burden of the consumer and increases his disposable income. Similarly, if $X < 0$, then the consumer pays a larger tax, which reduces his income. We define a new function, $w(s)$, which denotes the consumer's welfare as a function of s:

$$w(s) = E[u(A + s\bar{\theta} + sX)] \tag{5.1.1}$$

Differentiating this, we get

$$w'(s) = E[u'(A + s\bar{\theta} + sX)(\bar{\theta} + X)] \tag{5.1.2}$$

Since A is independent of X and $E(X) = 0$, we get $E[u'(A)X] = E[u'(A)]E[X] = 0$, so that the welfare functions derivative valued at $s = 0$ is

$$w'(0) = E[u'(A)(\bar{\theta} + X)] = \bar{\theta}E[u'(A)] + 0$$

$$w'(0) = \bar{\theta}E[u'(A)]. \tag{5.1.3}$$

Using the definition of a derivative, we can write

$$\lim_{s\to 0} \frac{E[u(A + s\bar{\theta} + sX) - u(A)]}{s} = \bar{\theta}E[u'(A)]. \tag{5.1.4}$$

Now, since we defined above the tax rate to be $s = 1/n \Leftrightarrow n = 1/s$, we can rewrite the above expression as

$$\lim_{n\to\infty} nE[u(A + \frac{\bar{\theta} + X}{n}) - u(A)] = \bar{\theta}E[u'(A)]. \tag{5.1.5}$$

Since consumer i is a risk averter, there exists a number $k(n) > 0$, so that the individual is indifferent between paying it, and facing the risk $(1/n)X$. $k(n)$ is the individual cost of risk bearing associated with the investment, i.e. the individual's risk premium. $k\,(n)$ thus satisfies

$$E\left[u\left(A + \frac{\bar{\theta} + X}{n}\right)\right] = E\left[u\left(A + \frac{\bar{\theta}}{n} - k\,(n)\right)\right]. \tag{5.1.6}$$

This leads to the insight, formulated in the following proposition.

Proposition 3. *When the size of the population tends to infinity, the share of an investment held by an individual goes to zero, and so too does the 'cost' of holding the risky asset, i.e.* $\lim_{n\to\infty} k(n) = 0$.

Proof. The proof to the above proposition is trivially that, if $k\,(n) = -X/n$, then $\lim_{n\to\infty} -X/n = 0$, and thus $\lim_{n\to\infty} k\,(n) = 0$. □

Additionally, the following stronger result can be shown.

Proposition 4. *The total of risk premiums for all individuals, $nk(n)$, approaches zero, as the number of taxpayers, n, gets large.*

Proof. Using the expressions (5.3.4) and (5.1.6) above we get

$$\lim_{n\to\infty} nE[u(A + \frac{\bar{\theta}}{n} - k(n)) - u(A)] = \bar{\theta}E[u'(A)] \tag{5.1.7}$$

where $\bar{\theta}/n - k(n) \to 0$ when $n \to \infty$. Using again the definition of a derivative, $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \Rightarrow f(x+h) \approx f(x) + hf'(x)$, we write

$$\lim_{n\to\infty} \frac{E\left[u\left(A + \frac{\bar{\theta}}{n} - k(n)\right) - u(A)\right]}{\frac{\bar{\theta}}{n} - k(n)} = E\left[u'(A)\right] > 0. \tag{5.1.8}$$

Dividing (5.1.8) with (5.1.7), we get

$$\lim_{n\to\infty} \frac{E\left[u\left(A + \frac{\bar{\theta}}{n} - k(n)\right) - u(A)\right]}{nE\left[u\left(A + \frac{\bar{\theta}}{n} - k(n)\right) - u(A)\right]\left(\frac{\bar{\theta}}{n} - k(n)\right)} = \frac{\bar{\theta}E[u'(A)]}{E[u'(A)]}$$

$$\lim_{n\to\infty} \left(\bar{\theta} - nk\left(n\right)\right)^{-1} = \bar{\theta}^{-1}$$

$$\lim_{n\to\infty} \bar{\theta} - nk(n) = \bar{\theta}$$

$$\lim_{n\to\infty} nk(n) = 0. \tag{5.1.9}$$

□

Proposition 5. *When n tends to infinity, the expected value of the project's net benefits, $E(\theta) = \bar{\theta}$, approximates the 'correct' measure of net benefits defined in terms of willingness-to-pay for an asset with an uncertain return.*

As noted by Arrow and Lind, the assumptions made in the above model about the identical nature of the individuals are not necessary for the results to hold. The basic intuitition is, that as the size of the population becomes large enough, the risk share of the public investment borne by any one individual becomes arbitrarily small. This conclusion leads to the unanswered question of how large does n need to be, to justify negliging the risk in a public investment. Another caveat to keep in mind is the initial assumption, that public investments are independent of each other, as clearly this is not always the case in reality.

The model leads to situations, where risky investments in the private sector are displaced by public investment with lower expected returns but higher returns when appropriately adjusted for risks, leading to a Hicks-Kaldor improvement. In other words, the government could more than pay the opportunity costs of the project to private investors.

Arrow and Lind conclude on a rather self-evident note, that the distribution of risk-bearing is crucial in deciding whether to invest, also outside the public sector, i.e. public companies with a wide owner base could use the same reasoning as the government. The use of the Arrow-Lind theorem in the context of private firms has been studied by, for example Bednarek and Moszoro (2014). Contrary to Hirshleifer, who suggested direct subsidies from the government to firms to incentivize risky investments, Arrow and Lind suggest the creation of adequate insurance markets as a solution.

Finally, the authors consider the possibility of non-identical taxpayers, so that the benefits of a certain public investment would accrue to a given part of the population, which leads to similar results as varying the degree of rivalry of the benefits and costs of the project. This would change the conclusions a bit from those already mentioned, in that a distinction should be made between public and private costs and benefits. This distinction examined in the following sections, where, following Fisher (1973), an environmental externality, or 'public bad' is first introduced to the basic model, and then by examining the effects of varying degrees of rivalry on the risk premia under different risk aversion profiles following Gallagher and Snow (2014).

The original paper's conclusions are known as the Arrow-Lind theorem, which states that a discount factor appropriate for certain returns should be used when calculating the present value of public costs and benefits where as a discount rate adjusted for risk should be used when assessing private costs and benefits. This result is problematized in the following extensions of the above basic model.

5.2 Model with Environmental Externalities

In the basic model, it was assumed that the costs and benefits from the public investment were private goods or bads. In fact, as will be seen from the results of this extension of the basic model, and from the further extension in 5.3, the results of the Arrow-Lind theorem critically hinge upon this very assumption. Following Fisher (1973) an environmental externality is now introduced to the basic model presented in 5.

We thus define, that the public investment not only yields the return θ/n to each consumer, but additionally produces a 'public bad', an environmental externality $C(m)$, that affects m individuals. Each of the suffering consumers in m is identically inflicted by the externality by the amount $C(m)/m$. We further define that the total amount of the public bad is increasing in m, i.e. $\partial C(m)/\partial m > 0$. The per capita negative effects might thus also grow in m, or just remain constant, and we have the condition, that $\lim_{m\to\infty} \frac{C(m)}{m} \neq 0$.

The consumers' reactions to the public investment are still captured by the same welfare function, w, as previously. However, as the population is now divided into two groups, they each have their own inputs into w. There are n consumers who gain equally from the net benefit θ, and m who suffer equally from the environmental externality. Since m want to be at least as well off after the investment as before it, an efficient investment decision is thus such, that it maximizes

$$nE[u(A + \frac{\theta}{n} - \frac{\phi}{n})] \tag{5.2.1}$$

subject to constraint

$$mE[u(A - \frac{C(m)}{m} + \frac{\phi}{m})] \geq mE[u(A)], \tag{5.2.2}$$

where ϕ is the total compensation that could be transeferred from the n gainers to the m losers. Using similar division as the one used on the benefits in the original model, $C(m)$ is divided into two parts, the certain costs $\overline{C(m)}$, and $Z(m)$ a zero mean random element. The constraint in (5.2.2) can now be written as

$$E[u(A - \frac{\overline{C(m)}}{m} - \frac{Z(m)}{m} + \frac{\phi}{m})] \geq E[u(A)], \tag{5.2.3}$$

dropping the ms outside the expectation operator for simplicity.

We now define a new risk premium associated with the externality. Assuming that the sufferer is risk averse, there exists some positive amount $l(m)$, such that the consumer would be indifferent between paying it and accepting the random element of the externality, $Z(m)$. $l(m)$ must thus satisfy

$$E[u(A - \frac{\overline{C(m)}}{m} - \frac{Z(m)}{m} + \frac{\phi}{m})] = E[u(A - \frac{\overline{C(m)}}{m} - l(m) + \frac{\phi}{m})]. \tag{5.2.4}$$

Plugging this into (5.2.3) and solving for ϕ, we get

$$E[u(A - \frac{\overline{C(m)}}{m} - l(m) + \frac{\phi}{m})] \geq E[u(A)]$$

$$E\left[u(A)\right] - E\left[u(A)\right] + \frac{\phi}{m} \geq \frac{\overline{C(m)}}{m} + l(m)$$

$$\frac{\phi}{m} \geq \frac{\overline{C(m)}}{m} + l(m)$$

$$\phi \geq \overline{C(m)} + ml(m). \tag{5.2.5}$$

Since we wanted to maximise (5.2.1), ϕ should be minimised, and thus (5.2.5) should hold with equality:

$$\phi = \overline{C(m)} + ml(m). \tag{5.2.6}$$

For the investment to be worthwhile, the usual net benefit criterion is applied, and the net benefits should exceed the costs of the externality. This leads to a minimum condition on whether or not the investment should be carried out:

$$\theta - \left(\overline{C(m)} + ml(m)\right) > 0. \tag{5.2.7}$$

If we now assume the consumers to be risk neutral, then we have $l\,(m) = 0$. In this case the results of the basic model in 5.1 hold, and the project should be carried out if the expected value of benefits net of all costs, now including also $\overline{C\,(m)}$, exceeds zero. If, however, the consumers are risk averse, and hence the sufferers' risk premia for the externality, $l(m)$, exist, then the results of the basic model need to be revised so long as the costs, the environmental externalities are significant. The expected costs of the externality as well as the compensation ϕ should then be added to the costs of the project. Thus, the project is now a Hicks-Kaldor improvement, but the compensations should actually be carried out for the investment to be worthwhile, from a societal perspective.

Generalizing on the last result by writing out the definition of the net benefits θ, from the initial model in 5.1, including the random return from the investment, X, and the associated risk premium $k\,(n)$, we write

$$E\left[u(A + \bar{\theta} + X - \phi)\right] = E\left[u\left(A + \bar{\theta} - k\,(n) - \left(\overline{C(m)} + ml(m)\right)\right)\right], \tag{5.2.8}$$

which denotes the consumer's expected utility from the pre-investment endowment A, expected investment benefits $\bar{\theta}$ net of the environmental costs ϕ and the mean-zero random investment return X. $k\,(n)$ is defined just as in the basic model, as the positive amount that the risk-averse consumer would pay that would leave him indifferent between paying and accepting the risk represented by X. We can now write the following proposition.

Proposition 6. *If the consumers' preferences exhibit risk aversion, and the proposed public investment yields an externality $C(m)$ which affects the group m, then the net benefit criterion for the acceptance of the project becomes $\bar{\theta} - k(n) - \left(\overline{C(m)} + ml(m)\right) > 0$.*

For the results derived above to matter, the environmental externalities must be significant to the affected consumers. Fisher mentions irreversible and cumulative environmental effects as such, which could most likely be non-negligible. If an insurance market was created which could internalise the environmental externalities, i.e. make possible the transfer of the risks from the sufferers to the society at large, then the benefits of risk-spreading, as formulated in the Arrow-Lind theorem, could be used in the case of externalities as well. The existence of insurance markets for environmental externalities are, however, unlikely in the real world, and thus, the implications a public bad has on the original model's results should be noted.

It would be desirable to internalise the environmental externalities of public projects, because this would correct a clear market failure, and eliminate the deadweight loss of individuals bearing the costs of the externalities, instead of the whole society. Internalisation should occur by assigning monetary values to the externalities, which, in the confines of this model, means that both the expected costs of the externality as well as the risk premium of the sufferers, together making up the Hicks-Kaldor compensation, should be accounted for in the decision criterion. This attempt to internalise the environmental externalities, that is, of trying to widen the risk-bearing base, can itself be costly – at times even more costly than the costs borne by the sufferers.

5.3 Model with Varying Degrees of Rivalry

As pointed out by Fisher (1973) and Foldes and Rees (1977), the results of the Arrow-Lind theorem do not seem to hold when the public investment yields benefits (or costs) that are public goods. In 5.2 a 'public bad' was introduced to the basic framework, but no comment was made as to the proportion of the benefits that are public and private. Public investments can, of course, have both rival and non-rival costs and benefits. This idea is now integrated into the basic model, following Gallagher and Snow (2014), by the coefficient α, $0 \leq \alpha \leq 1$, which denotes the proportion of benefits that are rival. Accordingly, $1-\alpha$ of the benefits are non-rival. Non-rivalry is here defined as one of the two basic properties of a public good, the other being non-excludability. The environmental externality $C(m)$ of the previous section can be now thought of as the fraction $1-\alpha$ of the projects 'benefits', that are non-rival. Benefits here can mean both benefits and costs, since costs can be thought of as negative benefits. Similarly, the net benefits θ were assumed previously as α, the rival fraction of the projects benefits.

Like in the basic model, the economy consists of n identical consumers and there is only one commodity. Each consumer receives a certain income A, the mean-zero random component of which, ε, can again be abstracted away. Though Gollier and Pratt (1996) find that the presence of an independent background risk, such as ε in this model, leads to increased risk aversion due to an effect called risk vulnerability. The public project produces the expected benefit $\bar{\theta}$ and a random return X. Each consumer finances the public project by paying a lump-sum tax, which is now defined differently from the basic model, as a fraction $\frac{\xi}{n}$ of the total costs of the project, ξ. Each consumer derives utility from both the rival and non-rival parts of the benefits of the project. The random rival and non-rival benefits are assumed to be perfectly correllated, i.e. an increase in a given project's private benefits also leads to an increase of the same degree in the public benefits.

The net benefits from the proposed public investment to each consumer is given by

$$B(\theta,\alpha,n) \equiv \theta(1-\alpha) + \frac{\theta\alpha-\xi}{n} \equiv \omega\theta - \frac{\xi}{n}. \tag{5.3.1}$$

Here the first term denotes the non-rival benefits from the public project and the second term denotes the share of the rival benefits, which is equal for all consumers. An additional coefficient is defined as $\omega = 1-\alpha(1-n)/n$. The expected utility of the consumer is given by

$$E_\theta\left[u\left(A+B(\theta,\alpha,n)\right)\right], \tag{5.3.2}$$

where u is the risk averting consumer's strictly concave von Neumann-Morgenstern utility function. Defining $\tilde{W} = A - \xi/n$, the consumer's net income, we get a new expression for the risk premium related to the project for each consumer, $k\left(\omega, \tilde{W}\right)$. The individual risk premium is defined implicitly in

$$E_\theta\left[u\left(\tilde{W} + (\theta - \xi)/n\right)\right] = u\left(\tilde{W} + \omega\bar{\theta} - k\left(\omega, \tilde{W}\right)\right). \tag{5.3.3}$$

We now look at the extreme cases, $\alpha = 1$ and $\alpha = 0$, where the project's benefits are either wholly rival or non-rival. We get, by using the definitions and equations thus far

$$E_\theta\left[u\left(\tilde{W} + (\theta - \xi)/n\right)\right] = u\left(\tilde{W} + \frac{\bar{\theta} - \xi}{n} - k\left(1/n, \tilde{W}\right)\right) \tag{5.3.4}$$

and

$$E_\theta\left[u\left(\tilde{W} + \theta - \xi/n\right)\right] = u\left(\tilde{W} + \bar{\theta} - \frac{\xi}{n} - k\left(1, \tilde{W}\right)\right) \tag{5.3.5}$$

for $\alpha = 1$ and $\alpha = 0$, respectively. From (5.3.4) and (5.3.5) it can be seen, that the expected benefits from the non-rival case are larger than from the rival case, since $\frac{\bar{\theta}-\xi}{n} < \bar{\theta} - \frac{\xi}{n}$. The variation of the expected benefits to each individual around the mean is smaller for $\alpha = 1$. Under constant absolute risk aversion (CARA), where differences in expected net benefits have no effects on the size of the risk premia, the risk premium for $\alpha = 1$ is smaller than the risk premium for $\alpha = 0$, since in the former case, only the private risk needs to be borne by a single consumer, i.e. we have $k\left(1/n, \tilde{W}\right) < k\left(1, \tilde{W}\right)$.

Since an increase in α means an increase in the project's rivalry, and it leads to a decrease in the coefficient ω, accordingly a decrease in α leads to a decrease in ω. Thus an increase in rivalry reduces the social cost of project risk if and only if k increases as ω increases. Differentiating equation (5.3.3) with respect to ω gives

$$E_\theta\left[u'\left(\left(\tilde{W} + \omega\theta\right) \cdot \theta\right)\right] = u'\left(\tilde{W} + \omega\bar{\theta} - k\left(\omega, \tilde{W}\right)\right) \cdot \left(\bar{\theta} - k_\omega\right), \tag{5.3.6}$$

where k_ω is an expression for the partial derivative of the risk premium with respect to ω, and can be solved to be

$$k_\omega = \bar{\theta} - E_\theta\left[u'\left(\left(\tilde{W} + \omega\theta\right) \cdot \theta\right)\right] / u'\left(\tilde{W} + \omega\bar{\theta} - k\left(\omega, \tilde{W}\right)\right). \tag{5.3.7}$$

Equivalently, it can be stated, that the social cost of risk, $k\left(\omega, \tilde{W}\right)$, declines as project rivalry increases if

$$\frac{dk}{d\alpha} = \frac{dk}{d\omega}\frac{d\omega}{d\alpha} = k_\omega \cdot \frac{d\left(1-\alpha\left(n-1\right)/n\right)}{d\alpha} = -\frac{n-1}{n}k_\omega \tag{5.3.8}$$

is negative. Equations (5.3.7) and (5.3.8) can be used to derive the following rule.

Proposition 7. *The social risk premium increases in ω and decreases in α.*

Proof. The above proposition holds, i.e., the partial derivative k_ω is positive, and k_α is negative, if and only if

$$\begin{aligned} k_\omega &> 0 \\ \Leftrightarrow \bar{\theta} - E_\theta\left[u'\left(\tilde{W}+\omega\theta\right)\cdot\theta\right]/u'\left(\tilde{W}+\omega\bar{\theta}-k\left(\omega,\tilde{W}\right)\right) &> 0 \\ \bar{\theta} &> E_\theta\left[u'\left(\tilde{W}+\omega\theta\right)\cdot\theta\right]/u'\left(\tilde{W}+\omega\bar{\theta}-k\left(\omega,\tilde{W}\right)\right) \\ u'\left(\tilde{W}+\omega\bar{\theta}-k\left(\omega,\tilde{W}\right)\right)\cdot\bar{\theta} &> E_\theta\left[u'\left(\tilde{W}+\omega\theta\right)\cdot\theta\right]. \end{aligned} \tag{5.3.9}$$

□

Taking a covariance of the terms on the right hand side of the inequality, and remembering that the utility function u represents risk averse preferences, we can write

$$\begin{aligned} \operatorname{cov}\left(u'\left(\tilde{W}+\omega\theta\right),\theta\right) &= E_\theta\left[\left(u'-E_\theta u'\right)\left(\theta-\bar{\theta}\right)\right] \\ = E_\theta\left[u'\theta-u'\bar{\theta}-\theta E_\theta u'+\bar{\theta}E_\theta u'\right] &= E_\theta\left(u'\cdot\theta\right)-E_\theta\left(u'\right)\cdot\bar{\theta}-\bar{\theta}E_\theta u'+\bar{\theta}E_\theta u' \\ &= E_\theta\left(u'\cdot\theta\right)-E_\theta\left(u'\right)\cdot\bar{\theta} < 0. \end{aligned} \tag{5.3.10}$$

Thus, inequality (5.3.9) holds, if

$$u'\left(\tilde{W}+\omega\bar{\theta}-k\right) \geq u'\left(\tilde{W}+\omega\bar{\theta}-\psi\right), \tag{5.3.11}$$

where ψ is the Kimball (1990) prudence premium. Since u is a strictly concave utility function, representing risk aversion[1], then the inequality in (5.3.11) holds if and only if $k \geq \psi$. This means that u exhibits constant or increasing absolute risk aversion (CARA or IARA). This leads to the following result.

Proposition 8. *k_ω is positive and k_α is negative if u exhibits CARA or IARA.*

Correspondingly, the case when the social risk premium increases as rivalry increases is now addressed, and leads to decreasing absolute risk aversion (DARA). The mean-zero random element of the project's benefits, $X = \theta - \bar{\theta}$, is now introduced back to the formulation. Now, the inequality in (5.3.9) should not hold for us to achieve the desired result, and we get

$$\left[u'\left(\tilde{W} + \omega\bar{\theta} - k\right) - E_\theta\left(u'\left(\tilde{W} + \omega\theta\right)\right)\right] \cdot \bar{\theta} \leq 0$$

$$\begin{aligned} E_\theta\left[u'\left(\tilde{W} + \omega\theta\right) X\right] &\geq \left[u'\left(\tilde{W} + \omega\bar{\theta} - k\right) - E_\theta\left(u'\left(\tilde{W} + \omega\theta\right)\right)\right] \cdot \bar{\theta} \\ &= \left[u'\left(\tilde{W} + \omega\theta - k\right) - u'\left(\tilde{W} + \omega\theta - \psi\right)\right] \cdot \bar{\theta}. \end{aligned} \tag{5.3.12}$$

This inequality holds, if the prudence ψ exceeds the risk premium k by a wide enough margin, that is, if DARA is strong enough. This can be written as the next proposition.

Proposition 9. *k_ω is negative, and k_α is positive, if DARA is sufficiently strong.*

It is, however, also possible that, given certain conditions, DARA exhibiting utility functions would lead to the opposite result, i.e. that the inequality in (5.3.9) would hold. We define an auxiliary function

$$H(\theta) \equiv u'\left(\tilde{W} + \omega\theta\right) \cdot \theta. \tag{5.3.13}$$

Since $u'\left(\tilde{W} + \omega\bar{\theta} - k\right) \cdot \bar{\theta} > u'\left(\tilde{W} + \omega\bar{\theta}\right) \cdot \bar{\theta}$, and given that the slope of a utility function representing risk aversion is larger the closer we are to the origin, we can write

$$u'\left(\tilde{W} + \omega\bar{\theta} - k\right) \cdot \bar{\theta} > H\left(\bar{\theta}\right) \geq E_\theta\left(H(\theta)\right) = E_\theta\left[u'\left(\tilde{W} + \omega\theta\right) \cdot \theta\right] \tag{5.3.14}$$

[1] I.e. $u' > 0$ and $u'' < 0$.

as a sufficient condition for inequality (5.3.9) to hold. This condition holds, if H is concave. To see this, we write out the first and second derivatives of H as

$$H' = u' + u''\omega\theta$$

and

$$H'' = \frac{d}{d\theta}\left(u' + \omega\theta u''\right) = u''\omega + \omega u'' + \omega^2\theta u''' = \left(u'''\omega\theta + 2u''\right)\omega$$

$$= u''\omega\left[2 - \hat{P}\right], \tag{5.3.15}$$

where $\hat{P} = -\omega\theta u'''/u''$ is the index of partial relative prudence. For $\hat{P} = -bu''(a+b)$ to be less than or equal to two for all $b \geq 0$ and $a + b > 0$, it is necessary and sufficient that the index of relative prudence $P = -bu'''(b)/u''(b)$ be less than or equal to two for all $b \geq 0$. Thus, H is concave if $P \leq 2$, since a concave functions second derivative should be negative, which is the case, if (5.3.15) has a value below zero.

Proposition 10. *k_ω is positive, and k_α is negative, if $P \leq 2$.*

We can also write the following proposition.

Proposition 11. *The social value of a public project of uncertain value increases as the degree of rivalry increases.*

Proof. The right hand side of equation (5.3.3) represents the certainty equivalent of the project. To see how it changes, as rivalry increases, we differentiate it with respect to α:

$$\frac{d}{d\alpha}\left(\tilde{W} + \omega\bar{\theta} - k\left(\omega, \tilde{W}\right)\right) = -\bar{\theta}\frac{n-1}{n} + k_\omega\frac{n-1}{n}$$

$$= -\frac{n-1}{n}\left(\bar{\theta} - k_\omega\right). \tag{5.3.16}$$

Using the expression we earlier derived for k_ω and substituting it into the above equation, we get

$$-\frac{n-1}{n}\left(\bar{\theta} - k_\omega\right) = -\frac{n-1}{n}\left(\bar{\theta} - \left[\bar{\theta} - \frac{E_\theta\left(u' \cdot \theta\right)}{u'}\right]\right)$$

$$\frac{n-1}{n}\left(\bar{\theta}-k_{\omega}\right)=\frac{n-1}{n}\frac{E_{\theta}\left(u'\cdot\theta\right)}{\bar{u}'}, \tag{5.3.17}$$

which is positive, since all of the terms n, u', and θ are positive. Multiplying this expression with the size of the population, n, yields the change in the project's social value as a result of greater rivalry, $(n-1)\cdot E_{\theta}\left(u'\cdot\theta\right)/\bar{u}'$. □

To study the effects of a change in the size of the population, we differentiate the individual risk premium with respect to population size.

$$\frac{dk}{dn}=\left[-\alpha k_{\omega}+\xi k_{\tilde{W}}\right]/n^{2} \tag{5.3.18}$$

since the partial derivatives of ω and $\tilde{W}$ with respect to the population size are $\omega_{n}=-\alpha/n^{2}$ and $\tilde{W}_{n}=\xi/n^{2}$. Differentiating (5.3.3) for $\tilde{W}$ yields $E_{\theta}\left[u'\left(\tilde{W}+\omega\theta\right)\cdot\theta\right]=u'\left(\tilde{W}+\omega\bar{\theta}-k\right)(1-k_{\tilde{W}})$, from which an expression for the partial derivative of individual risk premium with respect to $\tilde{W}$ can be solved as

$$1-k_{\tilde{W}}=E_{\theta}\left[u'\left(\tilde{W}+\omega\theta\right)\cdot\theta\right]/u'\left(\tilde{W}+\omega\bar{\theta}-k\right)$$

$$k_{\tilde{W}}=1-E_{\theta}\left[u'\left(\tilde{W}+\omega\theta\right)\cdot\theta\right]/u'\left(\tilde{W}+\omega\bar{\theta}-k\right). \tag{5.3.19}$$

Using the definitions of the partial derivatives of private risk premium from equations (5.3.7) and(5.3.19) and plugging them into (5.3.18) gives us the expression

$$\frac{n^{2}dk}{dn}=-\alpha\left[\bar{\theta}-\frac{E_{\theta}\left(u'\theta\right)}{\bar{u}'}\right]+\xi\left[1-\frac{E_{\theta}\left(u'\right)}{\bar{u}'}\right]. \tag{5.3.20}$$

Here $\bar{u}'=u'\left(\tilde{W}+\omega\bar{\theta}-k\right)$. From this, it follows that a necessary and sufficient condition for the private cost of risk bearing to decline as the population increases is

$$\xi\left[1-\frac{E_{\theta}\left(u'\right)}{\bar{u}'}\right]<\alpha\left[\bar{\theta}-\frac{E_{\theta}\left(u'\theta\right)}{\bar{u}'}\right]=\alpha\bar{\theta}\left[1-\frac{E_{\theta}\left(u'\theta\right)}{\bar{u}'\bar{\theta}}\right]. \tag{5.3.21}$$

If $\alpha=0$, then the sign of dk/dn in (5.3.18) is the same as the sign of the expression $\xi k_{\tilde{W}}$. As can be seen from (5.3.19), $k_{\tilde{W}}$, and thus $\xi k_{\tilde{W}}$ is negative only if the consumer's preferences exhibit decreasing absolute risk aversion. The term multiplying $\alpha\bar{\theta}$ in (5.3.21) is positive, since given inequality (5.3.9), which is a condition for $dk/d\alpha < 0$, the denominator is larger than the numerator and thus the fraction $\frac{E_\theta(u'\theta)}{\bar{u}'\bar{\theta}}$ is less than unity. Under DARA and CARA preferences, the term multiplying the costs, ξ, in the condition (5.3.21), is non-positive. Using the above formulations, the following proposition can be written.

Proposition 12. *dk/dn is negative, i.e. the private cost of risk is lower the larger the population, if all benefits are non-rival and consumers exhibit DARA preferences, or, if some portion of benefits are rival, preferences exhibit CARA or DARA, and the cost of risk bearing declines with greater rivalry.*

Thus, from the above extension of the basic model, it can be concluded, that the predictions that both private and social cost of risk bearing decline as the degree of rivalry increases and as the size of population increases, do hold, as long as preferences do not exhibit strong DARA. To conclude, it can be summarised that the social cost of risk declines with greater rivalry when relative risk aversion is less than one, i.e. preferences are represented by a concave utility function, and relative prudence is less than two.

Chapter 6

Discussion & Conclusions

So far, I have not even mentioned numerical values for a believable Social Discount Rate, and thus a brief incursion into the world of empirics now follows, followed by concluding remarks.

A recent working paper by Drupp et al. (2015) presents the results of their survey on what they call 'experts of social discounting', to achieve numerical estimates for suitable SDRs for the long term. The survey presents as results the answers from 197 experts, in whose opinion the long-term SDR should be between 1% and 3%. The survey was built so, that it asked both the numerical value for SDR as well as values for the components of SDR, as defined by the Ramsey equation, and only a minority of the answers are in line with the result the Ramsey equation would give, given the parameter values from the experts themselves. It should be noted, that Drupp et al.'s survey used the no-uncertainty version of the Ramsey equation, and thus no estimates of a social risk premium was asked. Accordingly, the experts called for more comprehensive approaches to intergenerational discounting, which could, among other things, include the use of a social risk premium.

If the experts of social discounting suggest the use of SDRs in the range of 1% to 3%, and criticise the use of a simple, certain Ramsey equation, what are then the actual rates used in public investment decisions and how do they compare to the suggestion of academics? The Finnish State Treasury publishes an annual suggestion of the nominal discount rate to be used in project assessment, which is 0.2% for the year 2016 (Valtiokonttori, 2016). This discount rate must, however, be considered a rate fit for 'certain' returns. From the perspective of long term social discounting, the discount rates used by the Finnish Transport Agency are of more interest. In their manual on project assessment, Goebel et al. (2011) suggests using a SDR of 4%, which is made up of a certain return part (2%) and a social risk premium (2%). In the manual, it is even emphasised, that no unambiguous reference value for the social risk premium exists. Goebel et al. (2011) also mentions the discount rates used in a few comparison countries as varying from 3% to 6%, which are differentially composed of a certain rate and a risk premium. Sweden is cited as using a separate rate for economic costs and environmental costs.

As a second best solution, Cost-Benefit Analysis should be used as a decision rule, since the derivation of social welfare functions might prove too laborous. To account for risk in CBA, the introduction of a societal risk premium was suggested. The risk premium can be thought of as an alternative to the calculation of certainty equivalents based on expected utility. The use of CBA, SDR and a social risk premium in public investment decisions are thus reasonably well motivated.

The Ramsey equation was formally derived, to be used as a theoretical backbone for considerations of a risk premium. The results presented in chapter 4 of this thesis show, that a comparatively simple formula for the Social Discount Rate can be derived, so long as the social risk premium can be left out, on account of the Arrow-Lind theorem. Even though the theorem has been questioned in economic theory, as well as used in actual choices of SDRs (Baumstark and Gollier, 2014), by integrating the extensions to the original model based on Fisher (1973) and Gallagher and Snow (2014), I have shown that its results hold if (a) the Hicks-Kaldor compensations from beneficiaries to sufferers are carried out, or (b) if the individuals' degrees of relative risk aversion and relative prudence are small enough.

Given the originally hypothetical nature of the Hicks-Kaldor compensations, and the considerable effort it would take to estimate the individuals' willingness to pay in each separate case of 'public bad' that a given project creates, it is the latter result, which seems plausibe to be used in empirical applications. Though the estimation of individuals' risk aversion and prudence are challenging, just as the estimation of their utility functions, it is possible (Hall, 1988; Gollier, 2001). Thus, to answer one of the research questions set out at the beginning of this thesis, the choice of the SDR is irrelevant of the degree of rivalry of benefits, if individuals' exhibit concave utility functions and a relative prudence of less than two.

On a more general level, and to answer another of my research questions, a distinction is needed between choice rules of a SDR for different types of projects. These distinctions should include, at least: local vs. national or global projects, short-run vs. long-run projects. Additionally, depending on whether the section 5.3's results hold, i.e. whether or not the existence of the social risk premium is dependent on the rivalry of the project's benefits, a distinction should be made between project's, whose benefits (and costs) are public vs. those where they are private. One possible set of rules for the choosing of SDR is offered by Heal (2005), though his suggestions are partly in conflict with other prominent authors on the topic.

Heal proposes the use of a time-preference rate in the case of discounting the possible costs and benefits of climate policy aimed at mitigating the impacts of global warming. The Stern Review, for example, uses a version of the Ramsey equation, conflicting with Heal's proposal. Still another approach, the use of discount rates that decline over time, as suggested by Weitzman (2001, 2010), was left out of considerations in this thesis altogether. Additionally it should be noted that, if for example the Ramsey equation is chosen, then, as Stern's many critics have pointed out (e.g., Nordhaus, 2007; Weitzman, 2007; Sterner and Persson, 2008), the choice of the parameter values in the equation lead to wildly differing results.

To answer the research question on the impacts of an environmental externality on the discount rate, the distinction between long and short projects needs once again be utilised. In the case of shorter investments, when there is quite naturally less uncertainty on all of the costs and benefits, including the environmental externalities, I suggest internalising the externalities the 'traditional way' by assigning them monetary values just as any other cost. If the project has a long time scale, however, the adjustment of the risk premium, if one is used, to account for the externalities seems reasonable. An alternative, of course, would be the setting up of the Hicks-Kaldor compensation funds, but, as said, the realisation of such an insurance market seems highly unlikely.

To conclude, it was shown in this thesis, that public investment decisions are of both theoretical and practical interest. Furthermore, the seemingly trivial choice of a Social Discount Rate, and especially the inclusion of a social risk premium in its formula is anything but trivial. Given the relatively non-strict conditions for the Arrow-Lind theorem to hold, even with private benefits, the use of the simpler, risk premium free version of the Ramsey equation seems justifiable. Additionally, it can be concluded that even after decades of academic discussion no unanimity exists on the choice rule of SDR. A conflict seems also to ensue in the choice of the values of the discount rate, no matter what the 'choice rule' used. Suggestions for future research around the topic of social discounting would include the examination of the Arrow-Lind theorem in the case, when time-variant discount rates, such as Weitzman's gamma discounting, are used.

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Appendix A

Mathematical Appendix

A.1 Euler equation

Since, in equilibrium, the net marginal utility of the individual from period t must equal the net marginal utility from the following period we have the equation

$$\frac{u'(c_t)}{(1+\delta)^{t-1}} = \frac{(1+\rho)\,u'(c_{t+1})}{(1+\delta)^t}$$

which can be solved as follows.

$$\frac{u'(c_t)}{(1+\delta)^{t-1}} = \frac{(1+\rho)\,u'(c_{t+1})}{(1+\delta)^t} \quad \| \cdot (1+\delta)^t$$

$$u'(c_t)(1+\delta) = u'(c_{t+1})(1+\rho) \quad \| \div (1+\rho)$$

$$u'(c_t)\frac{1+\delta}{1+\rho} = u'(c_{t+1}) \quad \| \div u'(c_t)$$

$$\frac{1+\delta}{1+\rho} = \frac{u'(c_{t+1})}{u'(c_t)},$$

where the last expression is the expression of the Euler equation used in the thesis.

A.2 Taylor approximation of ρ around c_t when g_{t+1} is small

The rule for a first order Taylor approximation is $f(x) \approx f(x) + (x - x_0) \frac{\partial f}{\partial x}$. Using this for the expression for the equilibrium interest rate $\rho_t(c_t)$ around the point c_t, given that g_{t+1} is approximately zero, we get the following results.

$$\rho_t(c_t) \approx \rho_t(c_t) + (c_t - 0) \frac{\partial \rho_t(c_t)}{\partial c_t}$$

Now, since $g_{t+1} \approx 0$, $u'(c_{t+1}) \approx u'(c_t)$ and $u'(c_t)/u'(c_{t+1}) \approx u'(c_t)/u'(c_t) = 1$, and we get

$$\rho_t(c_t) \approx 1 \cdot (1+\delta) - 1 + c_t \frac{\partial \rho_t(c_t)}{\partial c_t}$$

$$= \delta + c_t \left[(1+\delta) \frac{u''(c_t) u'(c_t) - u'(c_t) u''(c_t)(1+g_{t+1})}{(u'(c_t))^2} \right]$$

$$= \delta + c_t (1+\delta) \left[\frac{u''(c_t)}{u'(c_t)} - (1+g_{t+1}) \frac{u''(c_t)}{u'(c_t)} \right]$$

$$= \delta + (1+\delta) c_t (1 - (1+g_{t+1})) \frac{u''(c_t)}{u'(c_t)}$$

$$= \delta + (1+\delta) g_{t+1} \frac{-c_t u''(c_t)}{u'(c_t)}.$$

Now, by noting that $\delta \cdot g_{t+1} \approx 0$, when g_{t+1} is very small, we can further reduce the expression above to yield

$$\rho_t(c_t) \approx \delta + g_{t+1} \frac{-c_t u''(c_t)}{u'(c_t)}.$$

Defining now the elasticity of marginal utility of consumption as $\gamma(c) = -cu''(c)/u'(c)$, we can write the above expression to the form

$$\rho_t(c_t) \approx \delta + g_{t+1} \gamma(c_t),$$

as used in chapter 4.

A.3 First part of Arrow-Lind model

In this Appendix, I present the results of the first half of the Arrow and Lind (1970) article. We have an economy with one commodity and n identical consumers. The model follows Hirshleifer's state-preference approach (Hirshleifer, 1965), and there are Z possible states of the world and T periods. π_{iz} denotes consumer i's subjective probability of the state z occuring. The consumers own claims, x_{izt}, for various amounts of the commodity at different points in time, given different states of the world. $\bar{x}_{izt}$ denotes the initial claim of i to the commodity in period $t+1$ if state z prevails. The consumers can trade claims for the commodity in different future periods, and all trading is done in the first period. Claims are bought using commodity units which are assigned to specific time periods, and which are contingent on the state of the world in the period in question. All claims are constructed similarly, in that $\bar{x}_{izt}$ pays one commodity unit only in period $t+1$, if state z occurs, and nothing in all other cases.

Prices for the claims are written as $p_{zt}\,(z = 1, \ldots, Z; t = 0, \ldots, T-1)$. After the trading, the consumer owns x_{izt}, which he will excercise when the time comes to provide for his consumption. $v_i\left(x_{i1,0}, \ldots x_{i1,T-1}, x_{i2,0}, \ldots, x_{iZ,T-1}\right)$ is the strictly concave von Neumann-Morgenstern utility function of consumer i if he receives claims x_{izt}. Thus, each individual maximises

$$V_i\left(x_{i1,0}, \ldots x_{i1,T-1}, x_{i2,0}, \ldots, x_{iZ,T-1}\right)$$

subject to the constraint

$$\sum_{t=0}^{T-1} \sum_{z=1}^{Z} p_{zt} x_{izt} = \sum_{t=0}^{T-1} \sum_{z=1}^{Z} p_{zt} \bar{x}_{izt}.$$

Given that V_i is a strictly quasi-concave von Neumann-Morgenstern utility function, and using the state-preference approach to intertemporal decision-making by Hirshleifer, functions $U_{iz}\,(z = 1, \ldots, Z)$ can be found such that

$$V_i = \sum_{z=1}^{Z} \pi_{iz} U_{iz}\left(x_{iz0}, x_{iz1}, \ldots, x_{iz,T-1}\right).$$

Each individual's utility, given a state of the world, is a function of her consumption at each point in time. Writing out the Lagrangian and solving for the equilibrium conditions we get

$$\mathcal{L}(\cdot) = \sum_{s=1}^{S} \pi_{is} U_{is}(x_{is0}, x_{is1}, \ldots, x_{is,Q-1}) + \lambda_i [\sum_{q=0}^{Q-1} \sum_{s=1}^{S} (p_{sq}\bar{x}_{isq} - p_{sq}x_{isq})]$$

$$\frac{\partial \mathcal{L}}{\partial x_{isq}} = \pi_{is} \sum_{s=1}^{S} \frac{\partial U_{is}}{\partial x_{isq}} - \lambda_i \sum_{s=1}^{S} \sum_{q=0}^{Q-1} p_{sq} = 0$$

$$\pi_{is} \sum_{s=1}^{S} \frac{\partial U_{is}}{\partial x_{isq}} = \lambda_i \sum_{s=1}^{S} \sum_{q=0}^{Q-1} p_{sq}$$

$$\pi_{is} \frac{\partial U_{is}}{\partial x_{isq}} = \lambda_i p_{sq} \qquad (i = 1, \ldots, I; s = 1, \ldots, S; q = 0, \ldots, Q-1) \tag{A.3.1}$$

From the above result it can be deduced, that the ratio between prices for claims in periods q and m with respective states s and r is:

$$\frac{\pi_{is} \frac{\partial U_{is}}{\partial x_{isq}}}{\pi_{ir} \frac{\partial U_{ir}}{\partial x_{irm}}} = \frac{\lambda_i p_{sq}}{\lambda_i p_{rm}}$$

$$\Leftrightarrow \frac{p_{sq}}{p_{rm}} = \frac{\pi_{is} \frac{\partial U_{is}}{\partial x_{isq}}}{\pi_{ir} \frac{\partial U_{ir}}{\partial x_{irm}}} \quad (i = 1, \ldots, I; r, s = 1, \ldots, S; m, q = 0, \ldots Q-1)$$

p_{sq} can be thought of as the present value, valued at period 0 when trading takes place, of one commodity at time q, given state s, in terms of a certain claim to one commodity unit. Thus, the discount rate for discounting returns occuring at time q, given state s, to the value of time zero, can be found from the expression $p_{sq=\frac{1}{1+r_{sq}}}$. Following the same reasoning, the value for a certain claim to one commodity unit at time q is

$$p_q = \sum_{s=1}^{S} p_{sq}$$

and the rate of discount approriate for a certain return from time q is

$$\frac{1}{1+r_q} = \sum_{s=1}^{S} \frac{1}{1+r_{sq}} = \sum_{s=1}^{S} p_{sq}.$$

Perfect markets for claims contingent on world states are assumed. The return to a given investment is defined as $h_{sq}(s = 1, \ldots, S; q = 0, \ldots, Q-1)$. Investments should be carried out, if

$$\sum_{s=1}^{S} \sum_{q=0}^{Q-1} h_{sq} p_{sq} > 0,$$

i.e. when the net return, or, net present value is positive. Using the above definition for p_{sq} we write

$$\sum_{s=1}^{S} \sum_{q=0}^{Q-1} \frac{h_{sq}}{1+r_{sq}} > 0$$

as the condition. So far, in this model the discount rate r_{sq} is specific to each time-state.

Further assumptions are made,

- h_{sq} is independent of the returns of the previous investment (i.e. no auto-correllation),
- h_{sq} is independent of the individual (VNM) utility functions,
- h_{sq} has a *objective* probability distribution.

Additionally, the states of the world are partiotioned into collectively exhaustive and mutually exclusive sets E_t, where the subscript t is such that, for all possible states s in any given set E_t, all utility functions U_{is} are identical for a given individual i. Additionally, another partition is made, F_u, so that the net return h_{sq} is the same for all s in F_u. E_t and F_u are independent of each other and F_u is idenical for all individuals. A subset E_{tu} belongs to both E_t and F_u.

Now s in the previous formulations is replaced with tu, though $U_{is} = U_{itu} = U_{it}$ and $x_{isq} = x_{ituq} = x_{itq}$. π_{it} is the subjective probability, assigned by individual i, of E_t and π_u is the objective, shared probability of F_u. The statistical independence between the two partitions can the be written as

$$\pi_{itu} = \pi_{it}\pi_u.$$

Then, the equilibrium condition derived from the Lagrangian can be rewritten as

$$\pi_{it}\pi_u \frac{\partial U_{it}}{\partial x_{itq}} = \lambda_i p_{tuq}.$$

$$(\pi_{it} \frac{\partial U_{it}}{\partial x_{itq}})/\lambda_i = \pi_u p_{tuq}$$

Since π_u neither p_{tuq} nor depends on the individual i, the same must hold for the left-hand side of the above equation. The left-hand side expression is also independent of u and can be denoted as μ_{tq}. This gives us the expression

$$p_{tuq} = \mu_{tq}\pi_u.$$

The above expression, and the fact that returns to investments are the same for all s in F_u, allow us to reformulate the minimum condition for investments as follows

$$\sum_{s=1}^{S}\sum_{q=0}^{Q-1} h_{sq}p_{sq} = \sum_{q=0}^{Q-1}\sum_{t}\sum_{u} h_{uq}p_{tuq} = \sum_{q=0}^{Q-1}\sum_{t}\sum_{u} h_{uq}\mu_{tq}\pi_u$$

$$= \sum_{q=0}^{Q-1}(\sum_{t}\mu_{tq})\sum_{u}\pi_u h_{uq}.$$

Using the notion, that $p_q = \sum_{s=1}^{S} p_{sq}$ we get

$$p_q = \sum_{s=1}^{S} p_{sq} = \sum_{t}\sum_{u} p_{tuq} = (\sum_{t}\mu_{tq})(\sum_{u}\pi_u) = \sum_{t}\mu_{tq}$$

since the probabilities π_u sum up to one. We then arrive at the formulation

$$\sum_{q=0}^{Q-1}\sum_{s=1}^{S} h_{sq}p_{sq} = \sum_{q=0}^{Q-1}(\sum_{t}\mu_{tq})\sum_{u}\pi_u h_{uq} = \sum_{q=0}^{Q-1}\frac{1}{1+r_q}\sum_{u}\pi_u h_{uq}.$$

This equation tells us that, given the independence and objectivity conditions, the present value of an investment equals the present value of returns in each period, discounted by the factor approriate for certain returns at that time, i.e. risk need not be taken into account in the discounting process in this modelling of the world.